THE

Practical Teacher

A MONTHLY EDUCATIONAL JOURNAL.

Edited by JOSEPH HUGHES.

'*Knowledge is proud that he has learned so much,*
Wisdom is humble that he knows no more.'—COWPER.

VOL. VII. No. 6. AUGUST, 1887. PRICE, 6D. POST FREE, 7½D.

Animal Physiology.

BY ARTHUR NEWSHOLME, M.D., AND C.S.S. (LOND.)

THE EYE.

HITHERTO we have been chiefly concerned with the eye as an optical instrument, arranged to throw images of external objects on a sensitive surface; the two chief conditions for which are transparency of the refracting media, and accuracy of focussing.

Some change is produced in the sensitive part of the retina, which results in the transmission of a message along the optic nerve to the centre for sight within the brain.

It has been shown that the outer segments of the retina contain a peculiar purplish colouring matter, known as *rhodopsin*, which rapidly bleaches on exposure to light, and regains its colour if kept in darkness in contact with its natural background. The chemical changes occurring in rhodopsin are an accompaniment of the vital changes resulting in the sensation of vision. The retina is not equally sensitive all over; the yellow spot (*macula lutea*), and especially its central depression (*fovea centralis*) being the most sensitive.

Origin of Optic Nerves.

In order to understand more clearly the intellectual part of vision, the connection of the optic nerves with the brain should be studied.

A reference to fig. 65 will show that the optic nerves leave the two orbits in a direction which brings them near each other on the under-surface of the brain. At this point (known as the *optic chiasma*) the nerves partially cross, the roots of the optic nerves then diverging to each half of the under-surface of the brain. Each root winds across the under-surface of the crus cerebri, and at its origin from the brain is divided into two bands, which arise from the optic thalamus, corpus quadrigeminum, and corpus geniculatum. The deepest origin is probably from the geniculate bodies, as destruction of these is followed by complete loss of vision. At the optic commissure or chiasma there are four sets of nerve fibres as indicated in fig. 75; (1) decussating fibres passing from one side of the brain to the opposite eye; (2) less numerous external fibres which pass from the optic tract to the eye of the same side; (3) scanty fibres, called inter-central, which pass from one optic tract to the other, and do not enter the eye; (4) fibres passing from one eye to the other, without having any connection with the optic tract.

Fig. 75.

Diagram of Decussation at Optic Commissure.

The decussation of the optic nerves explains the interesting cases in which from injury of one optic nerve or its point of origin in the brain, loss of vision has resulted in the outer half of the retina on the same side, and the inner half of the retina on the opposite side to that of the lesion.

Outward Projection of Sensation.

Before describing the physiology of erect and binocular vision a fundamental law of vision should be stated. An image is formed on the retinal screen; but we do not see the retinal image; an impression is conveyed to the brain, and the brain refers this impression outward into space as an external image, the facsimile of the object which produced it. This law of *outward projection* is really an extension of a general law of sensation, in virtue of which stimulation in any part of a sensory nerve is referred to its peripheral extremity. Thus pressure on the sciatic nerve by sitting in an uncomfortable position, is followed by the abnormal sensation known as 'pins and needles' in the feet and toes. In tasting, the sensation is referred to the tongue, in which the gustatory nerve terminates. In smelling there is still a sensation in the nose, but there is a mental reference to an external body at a distance as the cause. In the case of hearing and seeing there is little or no subjective sensation, and the impressions are referred entirely to their external causes.

BRITISH MUSEUM

If any part of the optic nerve is irritated (as by an electric current), no pain is felt, but a flash of light is seen, not in the eye, but beyond in the field of view.

Press the finger into the inner corner of the eye; a brilliant coloured spectrum is seen on the opposite or external part of the field of view. The coloured spectrum is called a *phosphene*. It is caused by the pressure on the sclerotic, causing an irritation of the contiguous part of the retina; the impression produced being referred directly outward at a right angle to the point irritated.

It is evident from this experiment (and the result may be varied by altering the point of pressure on the eye) that every point in the retina has its invariable correspondent in the field of vision, and *vice versa*. The rays of light cross each other in the crystalline lens, and consequently not only are the retinal images inverted, but the relative positions of these images are inverted; so that the left half of the retina corresponds to the right half of the visual field, the upper half of the retina to the lower half of the field, and so on.

Erect Vision.

This goes far to answer the question, how is it that retinal images being all inverted, and external images being outward projections of retinal images, we see objects in their right position, *i.e.*, erect. The rays of light from any external object are mentally referred back along the line of vision into space, and therefore to their proper place. Thus the rays of light from a star to the extreme right of the field of vision, enter the eye, and strike the retina on its extreme left margin, whence they are referred straight back and the object seen in its proper place on the right. The same law is exercised when with eyes bandaged a person has a rod pushed against him, and he determines, by the direction of the blow, the point from which it was directed.

The common explanation of erect vision is that it is the result of education corrected by experience; but the physical explanation stated above quite explains the phenomenon.

Binocular Vision.

The two eyes are not to be considered as mere duplicates, supplied in case of the accidental loss of one. They act together as one instrument; and many visual phenomena, and the judgments founded on them, result entirely from this. An image is formed on each retina, and yet each object is appreciated as a unit.

The fact that double images of external objects are formed may be easily proved by the following experiment: Hold a small lead-pencil at a distance of a few inches from the eyes. It is distinctly seen as a single object. Next hold in the same line, a few inches further removed, another small object. When the first is distinctly seen, the second appears double; while if the eyes are fixed on the second, the first appears double.

It is evident therefore that when we look directly at anything we see it single, but that all objects nearer or more remote are seen double. The reason why the duplicity of external objects is not consciously perceived, is that we do not easily separate what we know from what we see. Consciousness interferes with the double image, and one image is ignored, although by experiments such as the preceding, it may be demonstrated.

When the external object is seen single, the two retinal images fall on *corresponding parts* of the two retinæ; and this happens when, if the two images are projected outward, they reach the same point in space and are therefore superimposed and coincide.

The existence of corresponding points on the two retinæ may be better understood by the statement of an analogous case, illustrated by the following experiment, which is as old as Aristotle:—A marble held between the forefinger and middle finger is perceived as a single object, whereas if these two fingers be crossed, as in fig. 76, and the marble be rolled about on the table between the points of the crossed fingers, two marbles will be distinctly perceived.

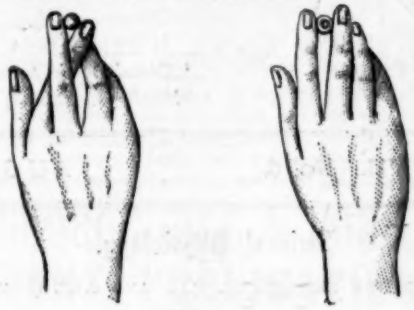


Fig. 76.
Aristotle's Experiment.

In recent or temporary squinting there is double vision, as non-corresponding parts of the two retinæ are stimulated by external objects.

The presence of corresponding points on the two retinæ is not confined to the yellow spot, but occurs throughout the surface of the two retinæ. This explains the formation of the *horopter*, *i. e.*, the surface of single vision.

In the experiment with the lead-pencil, an object nearer or further away from the point of vision is seen double, the explanation being that in these positions of the object the image does not fall on corresponding parts of the two retinæ. If however a lead pencil be held at a certain distance, and both eyes fixed on it, another pencil moved to one side or the other can be seen distinctly when it is carried in a certain curved line; but on either side of this line it appears double. This line or area (for the line may have any direction) is called the horopter. It is curved, but its precise form is unknown.

The perception of *direction* is a general property of sensory nerves, developed into mathematical accuracy in the case of the optic nerve; but the property of corresponding points in the two retinæ is less simple. It is probably not entirely the result of individual education, but largely of inherited experience. The chicken a few minutes after breaking the egg-shell, will see a morsel of food and unhesitatingly seize it, without any of the uncertainty which would be produced by double vision; and there are other instances of inherited powers even more wonderful than this.

All the phenomena of single vision may be explained by the *law of direction*, which explains the existence of corresponding retinal points, and points in external space: and all the phenomena of binocular vision can be explained by another *law of corresponding points*, which asserts a correspondence point for point of the two retinæ, and a coincidence of their representatives in the two fields of vision.

The importance of binocular vision may be understood by the bungling shown when an attempt is made to thread a needle with one eye shut. It is essential in obtaining a just appreciation of the size, form, and distance of external objects that both eyes should be used. The simple sense impressions on a single retina result only in the perception of *light*, its *intensity*, its *colour*, and its *direction*. These cannot be analysed into any simpler elements; and on them are based our judgments as to size, form, and distance.

Judgment of Distance.

In every voluntary act of sight there are three adjustments:—(1) a *focal* adjustment, the ciliary muscle adjusting each lens so as to bring the rays of light to a focus on the retina. (2) An *axial* adjustment, the two eyes being turned by the recti muscles, so that their axes shall meet at the point of sight, and the image of the object shall fall on the central spot of each eye. The first adjustment is necessary for *distinct* vision, the second for *single* vision. (3) An *adjustment of the pupil* accompanies the two preceding adjustments, the pupil contracting for near vision, so as to cut off the peripheral rays of light. These three adjustments occur without any separate volition for each; the single voluntary act of looking, is accompanied by them all.

We judge of distance to a large extent by the amount of muscular effort necessary to effect focal and axial adjustments. (a) Judgments founded on the efforts required for focal adjustment are fairly accurate from 5 inches to 20 feet, but beyond this the effort is so slight as to be inappreciable. (b) The greater or less effort of the internal recti muscles required to produce the amount of optic convergence necessary for single vision is a far more reliable mode of judging of distance; and it is reliable from near the root of the nose to the distance of 200 to 300 yards.

The man with only one eye cannot judge by this method; if he is dipping a pen into an inkstand or putting a stopper into a bottle, mistakes are apt to occur. If anyone tries to plunge his finger rapidly into the open mouth of a bottle, having one eye shut during the experiment, he will most likely overreach or fall short.

(c) *Mathematical perspective* is a means of great accuracy in estimating distance, and almost without limit. Objects become smaller and nearer together, the more remote they are. Parallel lines of any kind seem to converge in the distance. The diminution in apparent size of known objects enables us to form, by comparison, very accurate estimates of distance.

(4) *Aerial perspective* is of some use in estimating distance. The atmosphere is neither perfectly transparent nor colourless, and therefore more distant objects become dimmer and bluer. If the atmosphere is exceptionally clear or obscure, we may misjudge the distance.

The artist can imitate aerial perspective in his picture by dulling the sharpness of outline, bluing the tinge, and diminishing the brightness of all objects in proportion to their supposed distance. He can even more perfectly imitate the mathematical perspective by diminishing the size of objects and the distance between them, as the background of his picture is approached. The focal and binocular or aerial perspective are however beyond his reach. The only instrument which can artificially imitate the latter is the stereoscope. The absence of aerial perspective is partially concealed when a painting is looked at with only one eye, and thus the resemblance to an external landscape is increased.

Judgment of Size.

The apparent size of an unknown object is indicated by the angle which the object subtends. Thus in fig. 77 it is evident that precisely the same image would be made by a small object at *e*, a larger object at *d*, and a still larger (A B) at *c*, each of these subtending the angle A o B at the nodal point of the

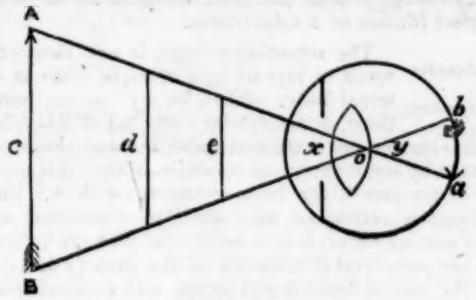


Fig. 77.

Visual Angle A o B; nodal point, o; *a b*, inverted image on retina. *c*, *d*, *e*, Varying apparent size of object according to distance.

retina. Therefore the estimated size of the object which produces an image on the retina, will depend on the distance we imagine the object to be from us. In other words the size of an unknown object is judged by the size of its retinal image multiplied by its estimated distance. The estimation of size and distance being thus closely associated, an error in one involves an error in the other. Thus, if the image of a man appears in one field of vision we infer if the image is small that the man is far off. It may be true that the reason of the smallness of the image is the distance of the man, in which case our estimate of size and distance is correct; or it may be that we have looked at the man, through some such arrangement as an inverted telescope, in which case our judgment is a delusion.

Judgment of Solidity.

When we look at any solid object at a short distance we see with the left eye a portion of the surface which is not seen with the right eye, and *vice versa*. Thus the impressions on the two retinæ are slightly dissimilar, but are fused in normal vision so as to produce the impression of a single object. If a truncated pyramid be placed in the middle line before us, the image projected on the left eye and on the right eye will differ, as seen in fig. 78, but the perception gained from the two images combined will correspond to the middle projection in fig. 78. It is by this combination of two dissimilar images for a single object, that we judge that the object giving rise to the images is solid.

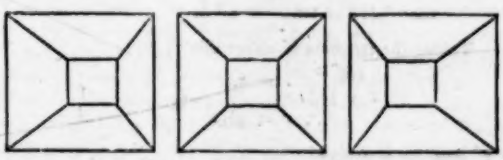


Fig. 78.

Projection of Truncated Pyramid seen with the two eyes.

Wheatstone first demonstrated, by means of the stereoscope, that the fusion of two dissimilar images, and consequent formation of a single image, gave the impression of solidity. In the stereoscope two pictures are made of a solid object, one viewed slightly from the right side, and the other slightly from the left, so as to imitate the difference in the images formed on the two retinæ. These pictures are so placed in a box that the image of one picture is formed on the right retina, and of the other on the left retina. When the conditions are accurately fulfilled, only a single image is seen, and this conveys to the mind the perfect illusion of a solid object.

Subjective Sensations.

The sensation of light is not always the result of rays of light received from an external body. When we say 'we see' something, in ninety-nine cases out of a hundred this is correct, but in the hundredth, the sensation is produced by some abnormal condition of the optic nerve, or of the part of the brain connected with it. Such subjective sensations may occur in connection with any sensory nerve, and in every case they are referred to the peripheral distribution of the sensory nerve, or in the case of hearing and seeing, into external space. The creeping and loathsome animals seen by a patient suffering from alcoholic delirium, are a common example of subjective sensations (also called *illusions*) of the sense of sight. When the patient who suffers from illusions of his senses, believes in their reality, he becomes the subject of intellectual *delusions*, and as such is insane.

The person who says 'he must believe his own senses' occasionally falls into error. What the senses testify in any given case is that they have been stimulated and excited. Whether this excitation has been caused by an abnomal condition of the nerve or of the nervous centre, or by an ordinary external sensory stimulus, they are not competent to decide. The mind must form its own judgment on the matter, aided by evidence from other senses than that already stimulated, and if necessary checked by the experience of other persons.

(To be continued.)

Easy Steps in Trigonometry.

BY REV. A. D. CAPEL, M.A.,

Author of 'Catch Questions in Arithmetic,' etc., etc.

ANSWERS AND HINTS TO SOLUTIONS.

CHAPTER XVIII.

1. $4\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{120}{119}$

$\tan^{-1}\frac{120}{119} - \tan^{-1}x = \frac{\pi}{4}.$

Taking the tangents of either side

$$\frac{\frac{120}{119} - x}{1 + \frac{120}{119}.x} = 1.$$

$$\therefore x = \tfrac{1}{239}.$$

2. Ans. 3·1415926536

The negative results, before multiplying by 41, being ·0148580644 and the positive before multiplying by 4 being ·8023622781.

3. Since $\tan^{-1}\frac{1}{239} = \tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$, they are the same.

4. $$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{5+2\sqrt{6}}{1}$$

and $$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\tan\left[\tan^{-1}(5+2\sqrt{6}) + \tan^{-1}\frac{\sqrt{6}}{2}\right]$$

$$= \frac{10+5\sqrt{6}}{-10-5\sqrt{6}} = -1 = \tan\frac{3\pi}{4}.$$

5. The series would be

$$\frac{3\pi}{4} = \left[(5+2\sqrt{6}) - \tfrac{1}{3}(5+2\sqrt{6})^3 + \tfrac{1}{5}(5+2\sqrt{6})^5 - \ldots\right]$$

$$+ \left[\frac{\sqrt{6}}{2} - \tfrac{1}{3}.\left(\frac{\sqrt{6}}{2}\right)^3 + \tfrac{1}{5}\left(\frac{\sqrt{6}}{2}\right)^5 - \ldots\right]$$

which would be very laborious to evaluate.

6. $$\left(1+\tan\frac{A}{2}\right)^2 = 1 + 2\tan\frac{A}{2} + \tan^2\frac{A}{2}$$

$$= \frac{\cos^2\frac{A}{2} + 2\sin\frac{A}{2}.\cos\frac{A}{2} + \sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}$$

$$= \frac{1+\sin A}{\frac{1+\cos A}{2}} = 2\frac{1+\sin A}{1+\cos A}$$

7. Draw a figure as in 54. Let rad. = 1.

$$\tan^2\frac{A}{2} = \frac{BN}{NP}.\frac{NP}{AN} = \frac{BN}{AN}$$

$$1+\tan^2\frac{A}{2} = 1 + \frac{BN}{AN} = \frac{2\text{ rad.}}{AN} = \frac{2}{AN}$$

$$2\tan\frac{A}{2} = \frac{2PN}{AN}$$

$$\therefore \left(1+\tan\frac{A}{2}\right)^2 = 2\frac{1+PN}{AN} = 2\frac{1+\sin A}{1+\cos A}$$

8. Reduce $\cos 3\phi$ and $\sin 3\phi$ by Forms. 43 & 45, whence we get $x = 4\cos^3\phi$ and $y = 4\sin^3\phi$.

Raise each equation to the $(\frac{1}{3})$rd power and add. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$. Ans.

9. $$\tan\sin^{-1}\frac{\sqrt{x}}{\sqrt{a+x}} = \frac{\sqrt{x}}{\sqrt{a}} \quad \therefore \text{ \&c.}$$

10. $$\text{Sin}\,2\sin^{-1}\frac{\sqrt{x}}{\sqrt{a+x}} = 2\frac{\sqrt{x}}{\sqrt{a+x}}.\frac{\sqrt{a}}{\sqrt{a+x}} = 2\frac{\sqrt{ax}}{\sqrt{a+x}}$$

$$\sin\cos^{-1}\frac{a-x}{a+x} = \frac{2\sqrt{ax}}{a+x}$$

$$\therefore 2\sin^{-1}\frac{\sqrt{x}}{\sqrt{a+x}} = \cos^{-1}\frac{a-x}{a+x} \quad \therefore \text{ \&c.}$$

11. $(\text{Cos}\,\alpha + \iota\sin\alpha)(\cos\beta + \iota\sin\beta)$

$= \cos\alpha\cos\beta + \iota^2\sin\alpha\sin\beta + \iota(\sin\alpha\cos\beta + \cos\alpha\sin\beta)$

remembering $\iota^2 = -1$

$= \cos(\alpha+\beta) + \iota\sin(\alpha+\beta).$

12. See 11.

13. Let there be n factors of the form. $(\cos\alpha + \iota\sin\alpha)$ all equal to one another.

$\therefore (\cos\alpha + \iota\sin\alpha)^n = (\cos n\alpha + \iota\sin n\alpha).$

14. $$\frac{1}{(\cos\alpha + \iota\sin\alpha)^n} = \frac{\cos^2 n\alpha + \sin^2 n\alpha}{\cos n\alpha + \iota\sin n\alpha}$$

$$= \frac{\cos^2 n\alpha - \iota^2\sin^2 n\alpha}{\cos n\alpha + \iota\sin n\alpha} = \cos n\alpha - \iota\sin n\alpha.$$

15. $\text{Cos } n\theta + \iota \sin n\theta = (\cos\theta + \iota \sin\theta)^n$

$= \cos^n\theta + n\cos^{n-1}\theta\,\iota\sin\theta + \frac{n(n-1)}{1.2}\cos^{n-2}\theta\,\iota^2\sin^2\theta + \frac{n(n-1)(n-2)}{1.2.3}\cos^{n-3}\theta\,\iota^3\sin^3\theta + \ldots$

$= \cos^n\theta\left[1 + n.\iota\tan\theta - \frac{n(n-1)}{1.2}\tan^2\theta - \frac{n(n-1)(n-2)}{1.2.3}\iota\tan^3\theta \ldots\right]$

16. In 15 equate possible and impossible parts

$$\cos n\theta = \cos^n\theta\left[1 - \frac{n(n-1)}{1.2}\tan^2\theta + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\tan^4\theta - \ldots\right]$$

$$\sin n\theta = \cos^n\theta\left[n\tan\theta - \frac{n(n-1)(n-2)}{1.2.3}\tan^3\theta + \ldots\right]$$

$$\tan n\theta = \frac{\sin n\theta}{\cos n\theta}$$

$$= \frac{n\tan\theta - \frac{n(n-1)(n-2)}{1.2.3}\tan^3\theta + \ldots}{1 - \frac{n(n-1)}{1.2}\tan^2\theta + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\tan^4\theta \ldots}$$

$$= n\tan\theta + \frac{(n+1)n(n-1)}{3}\tan^3\theta - \frac{(n+1)n(n-1)(n-2)(n-3)}{3.5}\tan^5\theta \ldots$$

17. Let $n\theta = a$. $\therefore n = \frac{a}{\theta}$.

$$\therefore \cos a = \cos^n\theta\left\{1 - \frac{\frac{a}{\theta}\left(\frac{a}{\theta}-1\right)}{1.2}\tan^2\theta + \frac{\frac{a}{\theta}\left(\frac{a}{\theta}-1\right)\left(\frac{a}{\theta}-2\right)\left(\frac{a}{\theta}-3\right)}{1.2.3.4}\tan^4\theta \ldots\right\}$$

$$= \cos^n\theta\left\{1 - \frac{a(a-\theta)}{1.2}\left(\frac{\tan\theta}{\theta}\right)^2 + \frac{a(a-\theta)(a-2\theta)(a-3\theta)}{1.2.3.4}\left(\frac{\tan\theta}{\theta}\right)^4 \ldots\right\}$$

18. If n be indefinitely increased and a remain the same θ becomes indefinitely diminished, but $\cos 0 = 1$ and $\frac{\tan\theta}{\theta}$, when θ is indefinitely diminished is also $= 1$ (see 217).

$$\therefore \cos a = 1 - \frac{a(a-1)}{1.2} + \frac{a(a-1)(a-2)(a-3)}{\lfloor 4} - \ldots$$

as we found in 224.

19. Apply Form. 1 to the sines in i. and ii., and obtain expressions for $\cos\theta$ and $\cos\phi$; hence find $\tan\theta$ and $\tan\phi$.

Substitute in iii. Ans. $\frac{1}{a} + \frac{1}{b} = \frac{1}{m} + \frac{1}{n}$.

20. It will be better to work in cosines and if we remove $\frac{\pi}{6}$ to L. H. M. and $\cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}}$ to R. H. M. we shall find it easier to prove, thus

$$\cos\left[\cos^{-1}\frac{\sqrt{2}}{\sqrt{3}} - \frac{\pi}{6}\right] = \frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\cdot\frac{1}{2} = \frac{\sqrt{6}+1}{2\sqrt{3}}$$

$$\therefore \cos^{-1}\frac{\sqrt{2}}{\sqrt{3}} - \frac{\pi}{6} = \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}}$$

21. $\tan\left[\cos^{-1}\frac{1-m^2}{1+m^2} - \cos^{-1}\frac{1-n^2}{1+n^2}\right]$

$$= \frac{\frac{2m}{1-m^2} - \frac{2n}{1-n^2}}{1 + \frac{4mn}{(1-m^2)(1-n^2)}} = \frac{2m - 2mn^2 - 2n + 2m^2n}{1 - m^2 - n^2 + m^2n^2 + 4mn} = \frac{2(m-n)(1+mn)}{(1+mn)^2 - (m-n)^2} \quad \therefore \text{ \&c.}$$

22. Reunite the denominator, thus, $(\cos a - \cos 3a) + (\sin 3a - \sin a)$, and apply Forms. H and D, whence the fraction becomes $\frac{1}{2\sin a}$ or $\frac{1}{2}$ cosec a.

23. Apply Form. E to both equations, when $\cos(\phi - 2\theta + 2a) = \cos(\phi - 2\theta - 2a)$ hence $\phi - 2\theta = 0$, and substituting this value in $\frac{1}{2}[\cos\phi + \cos(\phi - 2\theta + 2a)] = c$, we get

$$\cos\phi + \cos 2a = 2c.$$

24. From i. we can easily get $m = \frac{\cos^2\theta}{\sin\theta}$ and from ii. $n = \frac{\sin^2\theta}{\cos\theta}$

$$\therefore m^{\frac{2}{3}} + n^{\frac{2}{3}} = \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin^{\frac{2}{3}}\theta\,.\cos^{\frac{2}{3}}\theta}$$

$$\text{and } m^{\frac{2}{3}}n^{\frac{2}{3}} = \sin^{\frac{2}{3}}\theta\,.\cos^{\frac{2}{3}}\theta$$

$$\therefore (m^{\frac{2}{3}} + n^{\frac{2}{3}})\,m^{\frac{2}{3}}n^{\frac{2}{3}} = 1.$$

25. Drawing figure as in 7.

$$\cot^2\frac{A}{2} = \frac{AN}{NP}\cdot\frac{NP}{NB} = \frac{AN}{NB}$$

$$\frac{1+\cos A}{1-\cos A} = \frac{\text{rad.} + CN}{\text{rad.} - CN} = \frac{AN}{NB} \quad \therefore \text{ \&c.}$$

CHAPTER XIX.

1. The denominator of the result, which is given, suggests the factor, by which we should multiply each term, before we resolve it with the difference of two terms.

$$2\sin\tfrac{\phi}{2}\cos\theta = \sin\left(\theta + \tfrac{\phi}{2}\right) - \sin\left(\theta - \tfrac{\phi}{2}\right)$$

$$2\sin\tfrac{\phi}{2}\cos(\theta+\phi) = \sin\left(\theta + \tfrac{3\phi}{2}\right) - \sin\left(\theta + \tfrac{\phi}{2}\right)$$

$$\vdots \qquad\qquad \vdots$$

$$2\sin\tfrac{\phi}{2}\cos(\theta + \overline{n-1}\phi) = \sin\left(\theta + \tfrac{(2n-1)\phi}{2}\right) - \sin\left(\theta + \tfrac{(2n-3)\phi}{2}\right)$$

Adding and dividing by $2\sin\frac{\phi}{2}$,

$$\text{sum} = \frac{\sin\left(\theta + \frac{(2n-1)\phi}{2}\right) - \sin\left(\theta - \frac{\phi}{2}\right)}{2\sin\frac{\phi}{2}} = \frac{\cos\left(\theta + \frac{(n-1)\phi}{2}\right)\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \quad \text{by Form. D.}$$

2. If $\phi = \frac{2\pi}{n}$; $\sin\frac{n\phi}{2} = \sin\pi = 0$, whence the sum of the series $= 0$.

3. $\sin\left(\theta+\frac{\pi}{2}\right)=-\cos\theta$, $\sin(\theta+\phi)=\sin\left(\frac{\pi}{2}+\overline{\theta+\phi}\right)$
$=-\cos(\theta+\phi)$ &c., &c.

The sum of the series will be $\dfrac{\sin\left[\frac{\pi}{2}+\left(\theta+\frac{(n-1)\phi}{2}\right)\right]\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}$

which $=-\dfrac{\cos\left(\theta+\frac{(n-1)\phi}{2}\right)\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}$

whence changing all the signs we have the summation executed in 1.

4. In 261 for ϕ write θ and we have sum

$$=\frac{\sin\frac{(n+1)\theta}{2}\cdot\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$$

5. In 1 for ϕ write θ and we have sum

$$=\frac{\cos\frac{(n+1)\theta}{2}\cdot\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$$

6. If for ϕ in the series in 261 we write $\phi+\pi$, it becomes
$\sin\theta+\sin(\theta+\phi+\pi)+\sin(\theta+2\phi+2\pi)+$ &c.
which may be written thus
$\sin\theta-\sin(\theta+\phi)+\sin(\theta+2\phi)-\sin(\theta+3\phi)$
or the series we want to sum

$\therefore$ the sum of this is $\dfrac{\sin\left[\theta+\frac{(n-1)(\phi+\pi)}{2}\right]\sin\frac{n(\phi+\pi)}{2}}{\sin\frac{\phi+\pi}{2}}$

7. $\operatorname{Cosec}x=\dfrac{1}{\sin x}=\dfrac{\sin\frac{x}{2}}{\sin\frac{x}{2}\cdot\sin x}=\dfrac{\sin\left(x-\frac{x}{2}\right)}{\sin\frac{x}{2}\cdot\sin x}$

$$=\frac{\sin x\cos\frac{x}{2}-\cos x\sin\frac{x}{2}}{\sin\frac{x}{2}\cdot\sin x}$$

$$=\cot\frac{x}{2}-\cot x.$$

8. $\operatorname{Cosec}x=\cot\frac{x}{2}-\cot x$

$\operatorname{cosec}2x=\cot x-\cot 2x$
$\operatorname{cosec}4x=\cot 2x-\cot 4x$
$\operatorname{cosec}2^{n-1}x=\cot 2^{n-2}x-\cot 2^{n-1}x.$
Adding we have
$\text{sum}=\cot\frac{x}{2}-\cot 2^{n-1}x.$

9. $\tan x=\dfrac{\tan^2 x}{\tan x}=\dfrac{1-(1-\tan^2 x)}{\tan x}$
$=\dfrac{1}{\tan x}-\dfrac{2(1-\tan^2 x)}{2\tan x}$
$=\cot x-2\cot 2x.$

10. $\tan x=\cot x-2\cot 2x$
$\frac{1}{2}\tan\frac{x}{2}=\frac{1}{2}\cot\frac{x}{2}-\cot x$
$\frac{1}{2^2}\tan\frac{x}{2^2}=\frac{1}{2^2}\cot\frac{x}{2^2}-\frac{1}{2}\cot\frac{x}{2}$
$\vdots\qquad\qquad\vdots$
$\frac{1}{2^{n-1}}\tan\frac{x}{2^{n-1}}=\frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}}-\frac{1}{2^{n-2}}\cot\frac{x}{2^{n-2}}$
Adding we have the
$\text{sum}=\frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}}-2\cot 2x.$

11. $\sin\theta+\sin 3\theta+\ldots\ldots\sin(2n-1)\theta$
$=\dfrac{\sin[\theta+(n-1)\theta]\sin n\theta}{\sin\theta}$
and $\cos\theta+\cos 3\theta+\ldots\ldots\cos(2n-1)\theta$
$=\dfrac{\cos[\theta+(n-1)\theta]\sin n\theta}{\sin\theta}$

Dividing the one equality by the other we have the identity required.

12. By 6, writing θ for ϕ we have the sum of the numerator of Left Hand Member

$$\frac{\sin\left[\theta+\frac{(n-1)(\theta+\pi)}{2}\right]\sin\frac{n(\theta+\pi)}{2}}{\sin\frac{\theta+\pi}{2}}$$

and the first factor of the numerator will reduce to

$$\sin\left[\frac{2\theta+n\theta-\theta+n\pi-\pi}{2}\right]$$

and this only differs in sign from

$\sin\left[\frac{2\theta+n\theta-\theta+n\pi-\pi+2\pi}{2}\right]$ or $\sin\frac{(n+1)(\theta+\pi)}{2}$

similarly the first factor of the sum of the denominator of the Left Hand Member only differs in sign from

$\cos\left[\frac{2\theta+n\theta-\theta+n\pi-\pi+2\pi}{2}\right]$ or $\cos\frac{(n+1)(\theta+\pi)}{2}$

hence the identity.

13. By Form. E if S = sum.
$2S=[\cos(2\theta+\alpha)+\cos\alpha]+[\cos(2\theta+3\alpha)+\cos\alpha]$ to n terms
$=\cos(2\theta+\alpha)+\cos(2\theta+3\alpha)\ldots$ to n terms
$$=\frac{\cos\left(2\theta+\alpha+\frac{n-1}{2}\cdot 2\alpha\right)\sin\frac{n\cdot 2\alpha}{2}}{\sin\frac{2\alpha}{2}}+n\cos\alpha.$$

$$\therefore S=\frac{\cos(2\theta+n\alpha)\sin n\alpha}{2\sin\alpha}+\frac{n}{2}\cdot\cos\alpha.$$

14. If S = sum required
$2S=(\cos\alpha-\cos 3\alpha)+(\cos\alpha-\cos 5\alpha)\ldots$ to 6 terms
$=6\cos\alpha-(\cos 3\alpha+\cos 5\alpha)\ldots$ to 6 terms

$$\therefore S=3\cos\alpha-\frac{\cos(3\alpha+\frac{4}{2}\cdot 2\alpha)\sin\frac{6\cdot 2\alpha}{2}}{2\sin\frac{2\alpha}{2}}$$

$$=3\cos\alpha-\frac{\cos 8\alpha\sin 6\alpha}{2\sin\alpha}$$

15. The given result suggests the solution. In it let n = 1, and reduce thus

$$\tan\theta-\tan\frac{\theta}{2}=\frac{\sin\theta\cdot\cos\frac{\theta}{2}-\cos\theta\sin\frac{\theta}{2}}{\cos\theta\cdot\cos\frac{\theta}{2}}$$

$$=\frac{\sin\left(\theta-\frac{\theta}{2}\right)}{\cos\theta\cdot\cos\frac{\theta}{2}}=\tan\frac{\theta}{2}\sec\theta.$$

hence resolve each term into the difference of two terms and add r.

16. Hence again the result suggests the method of solution. In the result let n = 1.

$$\frac{\cot\theta-\cot 2\theta}{\sin\theta}=\frac{\sin 2\theta\cdot\cos\theta-\cos 2\theta\sin\theta}{\sin\theta\cdot\sin\theta\cdot\sin 2\theta}$$

$$=\frac{1}{\sin\theta\cdot\sin 2\theta}\text{ &c., &c.}$$

17. $\tan\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{2}\right)=\frac{1-\frac{1}{2}}{1+\frac{1}{2}}=\frac{2-1}{2+1}=\frac{1}{1+1+1^2}$

$\tan(\tan^{-1}\frac{1}{1}-\tan^{-1}\frac{1}{2})=\frac{\frac{1}{1}-\frac{1}{2}}{1+\frac{1}{2}}=\frac{1}{1+2+2^2}$

18. Ans. $\frac{\pi}{4} - \tan^{-1}\frac{1}{4}$.

19. In the result for 9 we write $(8 + 1)$, and then if for 8 we write 1, we get $\frac{\tan 2\theta - \tan\theta}{2\sin\theta}$ and this is

$$= \frac{\sin\theta}{2\sin\theta \cdot \cos 2\theta \cos\theta} = \frac{1}{2\cos 2\theta\cos\theta}$$

$$= \frac{1}{\cos\theta + \cos 3\theta}, \text{ whence the proof.}$$

20. $\operatorname{Cot}\left(\cot^{-1}\frac{a}{2} - \cot^{-1}a\right) = \frac{a \cdot \frac{a}{2} + 1}{a - \frac{a}{2}} = \frac{a^2 + 2}{a}$

$$\therefore \cot^{-1}\frac{a}{2} - \cot^{-1}a = \cot^{-1}\left(\frac{2}{a} + a\right).$$

21. Operate as in 19.

22. If $x^2 - 2px + q = 0$

$$x = p \pm \sqrt{p^2 - q}$$

$$= p\left\{1 \pm \sqrt{1 - \frac{q}{p^2}}\right\}$$

Let $\frac{q}{p^2} = \sin^2\theta$

$$x = p\,[1 \pm \sqrt{1 - \sin^2\theta}]$$

$$= p\,[1 \pm \cos\theta].$$

23. As before $x = p\left\{1 \pm \sqrt{1 - \frac{q}{p^2}}\right\}$

let $\frac{q}{p^2} = \sec^2\theta$

$$x = p\,[1 \pm \sqrt{-1}\sqrt{\sec^2\theta - 1}]$$

$$= p\,[1 \pm \sqrt{-1}\tan\theta].$$

24. As before we can easily show

$$x = p\left\{1 \pm \sqrt{1 + \frac{q}{p^2}}\right\}$$

Let $\frac{q}{p^2} = \tan^2\theta$

$$x = p\,[1 \pm \sqrt{1 + \tan^2\theta}]$$

$$= p\,[1 \pm \sec^2\theta]$$

$$= p\,\frac{\cos\theta \pm 1}{\cos\theta}$$

but $p^2\tan^2\theta = q$

or $p = \sqrt{q}\cot\theta$

$$\therefore x = \sqrt{q}\cdot\frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta \pm 1}{\cos\theta} = \sqrt{q}\,\frac{\cos\theta \pm 1}{\sin\theta}$$

$$= \sqrt{q}\cdot\frac{2\cos^2\frac{\theta}{2}}{2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}} \text{ or } -\sqrt{q}\,\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}$$

$$= \sqrt{q}\cot\frac{\theta}{2} \text{ or } -\sqrt{q}\tan\frac{\theta}{2}.$$

25. Ans. $x = 2;\ \sqrt{3} - 1;\ -\sqrt{3} - 1$.

CHAPTER XX.

DEMOIVRE'S THEOREM.

264. As Exercises to Chapter XVIII., viz., numbers 11, 12, 13, and 14, we practically proved this celebrated and important theorem for any integral value of the index, and in the exercises which followed these, we obtained from the theorem some important results, which we had, however, proved independently in an earlier chapter.

265. Demoivre's theorem is as follows:—

Cos $n\theta + \sqrt{-1}\sin n\theta$ is one of the values of $(\cos\theta + \sqrt{-1}\sin\theta)^n$, whatever be the value of n, positive or negative, integral or fractional.

266. The Student must notice that $\cos n\theta + \sqrt{-1}\sin n\theta$ is only one of the values of $(\cos\theta + \sqrt{-1}\sin\theta)^n$ but this theorem suggests how we may find the remaining values.

267. As an illustration the student probably has learnt that though $1^3 = 1$, there are two other expressions, involving impossible quantities it is true, which cubed will produce 1, viz., $\frac{1}{2}(-1 + \sqrt{-3})$ and $\frac{1}{2}(-1 - \sqrt{-3})$, as he can verify for himself.

268. The proof of Demoivre's theorem when it is a positive integer will be found as stated in 258, in the solutions to Chapter XVIII.

269. If n be a fraction, say $\frac{p}{q}$, we can prove it thus, writing ι for $\sqrt{-1}$, for θ write $\frac{p}{q}\theta$, and for n write q

$$\therefore \left(\cos\frac{p}{q}\theta + \iota\sin\frac{p}{q}\theta\right)^q = \cos p\theta + \iota\sin p\theta$$

$$= (\cos\theta + \iota\sin\theta)^p$$

taking the qth root on both sides

$$\cos\frac{p}{q}\theta + \iota\sin\frac{p}{q}\theta = (\cos\theta + \iota\sin\theta)^{\frac{p}{q}}$$

270. We also proved in the solutions to Chapter XVIII., that $(\cos\alpha + \iota\sin\alpha)^{-n} = \cos n\alpha - \iota\sin na$

but $\cos na = \cos(-na)$

and $-\iota\sin na = \iota\sin(-na)$

$\therefore$ we get $(\cos\alpha + \iota\sin\alpha)^{-n} = \cos(-na) + \iota\sin(-na)$

271. In 269 we might write $\frac{p}{q}\theta$ for θ, and $-q$ for q, and obtain $\cos\left(-\frac{p}{q}\theta\right) + \iota\sin\left(-\frac{p}{q}\theta\right)$

$$= (\cos\theta + \iota\sin\theta)^{-\frac{p}{q}}$$

hence the theorem is universally proved.

272. We thus find all the q values of $(\cos\theta + \iota\sin\theta)^{\frac{p}{q}}$ $\cos\theta$ and $\sin\theta$ remain unaltered if increased by any multiple of 4 right angles, that is

$\cos\theta = \cos\theta, \cos(\theta + 2\pi), \cos(\theta + 2.2\pi),$
$\cos(\theta + 3.2\pi)$, &c.

and $\sin\theta = \sin\theta, \sin(\theta + 2\pi), \sin(\theta + 2.2\pi),$
$\sin(\theta + 3.2\pi)$, &c.

or generally $\cos\theta = \cos(\theta + 2r\pi)$ and
$\sin\theta = \sin(\theta + 2r\pi)$

in which r may be 0, 1, 2, ... $(q - 1)$ as we will presently show.

$$(\cos\theta + \iota\sin\theta)^{\frac{p}{q}} = [\cos(2r\pi + \theta) + \iota\sin(2r\pi + \theta)]^{\frac{p}{q}}$$

$$= \cos\frac{p}{q}(2r\pi + \theta) + \iota\sin\frac{p}{q}(2r\pi + \theta)$$

If for r we write 0, 1, 2 ... $(q - 1)$, we obtain q different values for it, none of which are identical, but if for r we write q, or $(q + 1)$, &c., we should obtain a value we have had before.

273. The well known fact that there are n and only n roots to an equation of the nth degree would prove what is stated at the conclusion of the last article, but it can be proved independently thus.

274. If possible let $r = m$ and $r = n$ (when m and n are both less than q) give us identical values of the cosines and of the sines, then the angles $\frac{p}{q}(2m\pi+\theta)$ and $\frac{p}{q}(2n\pi+\theta)$ must differ by a multiple of 2π or $\frac{p}{q}(2m\pi+\theta) = \frac{p}{q}(2n\pi+\theta) = 2s\pi$

or $\frac{p}{q}[2m\pi+\theta \text{ or } 2n\pi+\theta] = 2s\pi$

or $\frac{p}{q}[2\pi(m \text{ or } n)] = 2s\pi$

or $\frac{p}{q}[m \text{ or } n] = s$ an integer, which since m and n are both less than q, and p is prime to q, is impossible.

275. To prove that if r be greater than $(q-1)$ we shall obtain a value we have had before.

Let $r = kq + t$, where t is positive and less than q and k may be any integer.

$$\cos\frac{p}{q}\left[2(kp+t)\pi+\theta\right] + \iota\sin\frac{p}{q}\left[2(kq+t)\pi+\theta\right]$$

$$\cos\left[2kp\pi+\frac{p}{q}(2t\pi+\theta)\right] + \iota\sin\left[2kp\pi+\frac{t}{q}(2t\pi+\theta)\right]$$

$$= \cos\frac{p}{q}(2t\pi+\theta) + \iota\sin\frac{p}{q}(2t\pi+\theta)$$

which is a value we have had before since t is less than q.

276. Hence of $(\cos\theta+\iota\sin\theta)^{\frac{p}{q}}$ we have these q values,

$$\cos\frac{p}{q}\theta + \iota\sin\frac{p}{q}\theta,\ \cos\frac{p(\theta+2\pi)}{q} + \iota\sin\frac{p(\theta+2\pi)}{q}$$

$$\ldots\ \cos\frac{p[\theta+2(q-1)\pi]}{q} + \iota\sin\frac{p[\theta+2(q-1)\pi]}{q}$$

277. In the collection of examples found at the end of this chapter, the student will find, besides a few in Demoivre's theorem, some miscellaneous ones which will test his knowledge of the whole subject.

278. EXAMPLES XX.

1. Find the three values of $(\cos\theta+\sqrt{-1}\sin\theta)^{\frac{1}{3}}$

2. Find the values of $(-1)^{\frac{1}{3}}$.

3. Extract the square root of $\cos 4A \pm \sqrt{-1}\sin 4A$.

4. Find the six values of $(-1)^{\frac{1}{3}}$.

5. Find the three values of $[1+\sqrt{-1}]^{\frac{1}{3}}$.

6. If $\tan\theta = \frac{b}{a}$, show by Demoivre's theorem that

$$(a+b\sqrt{-1})^{\frac{m}{n}} + (a-b\sqrt{-1})^{\frac{m}{n}} = 2(a^2+b^2)^{\frac{m}{2n}}\cos\frac{m}{n}\theta.$$

7. Given $\frac{\sin\theta}{\theta} = \frac{11}{12}$ show that θ is nearly the Circular Measure of 3°.

8. If $x = \cos\theta + \sqrt{-1}\sin\theta$
Find $x + x^{-1}$ and $x - x^{-1}$.

9. Using the notation of 8
Find $x^n + x^{-n}$ and $x^n - x^{-n}$.

10. Expand $(x+x^{-1})^n$ and thence prove

$$2^5\cos^6\theta = \cos 6\theta + 6\cos 4\theta + \frac{6.5}{1.2}\cos 2\theta + \frac{6.5.4}{2.(1.2.3)}$$

11. Prove $2^4\cos^5\theta = \cos 5\theta + 5.\cos 3\theta + \frac{5.4}{1.2}\cos\theta$.

12. Account for the extra 2 in the denominator of last term in 10 which is not found in last term in 11.

13. In any triangle ABC, AO, BO, CO are drawn bisecting the angles A, B, C, respectively.
If AO = l, BO = m, and CO = n, prove

$$l^2\left(\frac{1}{c}-\frac{1}{b}\right) + m^2\left(\frac{1}{a}-\frac{1}{c}\right) + n^2\left(\frac{1}{b}-\frac{1}{a}\right) = 0.$$

14. If in any triangle $2B = A + C$, prove
$2\cot\frac{1}{2}B = \cot\frac{1}{2}A + \cot\frac{1}{2}C$.

15. Show that $\sqrt{1+\sin\theta} = 1 + 2\sin\frac{1}{4}\theta\sqrt{1-\sin\frac{1}{2}\theta}$.

16. Give the perimeter and the three angles of a triangle, determine its side a.

17. A man ascends a mountain by a path, which is the shortest distance between the base and vertex. The inclination of his path to the horizon is at first θ, but afterwards it increases suddenly to ϕ and then remains unaltered. At the top he finds by the barometer that he has ascended m feet in vertical height and observes the angle of depression x of his starting-place. Find the distance travelled by him in the ascent.

18. Three circles whose radii are a, b, c, touch each other externally; prove that the tangents at the points of contact meet in a point whose distance from any one of them is

$$\sqrt{\frac{abc}{a+b+c}}.$$

19. Find θ from the equation
$\sin\theta - \cos\theta = 4\cos^2\theta\sin\theta$.

20. Find θ from the equation
$\sin(\alpha-\theta) = \cos(\alpha+\theta)$.

21. Prove $\cot^{-1}(1+1+1^2) + \cot^{-1}(1+2+2^2) + \ldots$ to 6 terms $= \tan^{-1}\frac{3}{4}$.

22. If $\sin^2 A + \sin^2 B = \sin^2 C$, prove that $A + B = C = 90°$.

23. Show that in any triangle the length of the line from C, bisecting the side c is $\frac{1}{2}\sqrt{2(a^2+b^2)-c^2}$.

24. Prove that $\tan^{-1}\frac{x\cos\phi}{1-\sin\phi} - \tan^{-1}\frac{x-\sin\phi}{\cos\phi} = \phi$.

25. Solve $\tan^{-1}\frac{1}{4} + 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} + \cot^{-1}x = \frac{1}{4}\pi$.

(To be continued.)

Christopher Columbus, by Lamartine.

TRANSLATED BY A. M. BOWER, F.R.G.S.,

Late Master in University College School, London.

CHRISTOPHE COLOMB. CHRISTOPHER COLUMBUS.

PREMIÈRE PARTIE. FIRST PART.

CHAPTER VIII.—*continued.*

Il espérait y trouver	He hoped to find there
des occasions et des moyens	opportunities and means
de s'élancer à son gré	of starting to his liking
sur l'Océan ;	on the Ocean ;
il n'y trouva que	he found there only
le travail ingrat	the unrequited labour
du géographe sédentaire,	of a sedentary geographer,
l'obscurité et l'amour.	obscurity and love.
En allant, chaque jour,	When going, each day,
assister aux offices religieux	to attend the religious services
dans l'église	in the church
d'un convent de Lisbonne,	of a convent at Lisbon
il s'éprit d'attachement	he became enamoured
pour une jeune recluse	of a young recluse (nun)
dont la beauté	whose beauty
l'avait frappé.	had smitten him.
C'était la fille	She was the daughter
d'un noble Italien	of an Italian cavalier,
attaché au service du Portugal.	attached to the service of Portugal.
Son père l'avait confiée	Her father had intrusted her
aux religieuses de ce convent	to the nuns of this convent
en partant pour	on setting out on
une expédition navale lointaine.	a distant naval expedition.
Elle s'appelait Doña	She was called Doña
Felippa de Palestrello.	Felippa de Palestrello.
Séduite elle-même	Attracted herself
par la beauté pensive et majestueuse	by the pensive and dignified beauty
du jeune étranger	of the young stranger
qu'elle voyait chaque jour	whom she saw every day
assidu aux services de l'église,	assiduous at the church services,
elle ressentit l'amour	she reciprocated the love
qu'elle lui avait inspiré.	with which she had inspired him.
Tous deux sans parents	Both being orphans
et sans fortune	and without fortune
sur une terre étrangère,	in a strange land,
rien ne pouvait contrarier	nothing could oppose
l'attrait qu'ils éprouvaient	the attraction which they felt
l'un pour l'autre ;	for each other ;
ils s'unirent par un mariage,	they were united in marriage
sur la foi de la Providence,	on the faith of Providence,
et du travail, seule dot	and of labour, the only dowry
de Felippa et de son amant.	of Felippa and of her lover.
Il continuait, pour nourrir	He continued, in order to support
sa belle-mère, sa femme et lui,	his mother-in-law, his wife and himself
à faire des cartes	to make charts
et des globes recherchés	and globes sought after
à cause de leur perfection	on account of their perfection
par les navigateurs portugais.	by the Portuguese navigators.
Les papiers de son beau-père,	The papers of his father-in-law,
qui lui furent remis	which were put into his hands
par sa femme,	by his wife
et ses correspondances	and his (own) correspondence
avec Toscanelli,	with Toscanelli,
fameux géographe de Florence,	a famous geographer of Florence,
lui fournirent, dit-on,	supplied him, it is said,
des notions précises,	with exact ideas,
sur les mers lointaines	about the distant seas
de l'Inde, et les moyens	of India, and the means
de rectifier les éléments	of correcting the elements
alors confus ou fabuleux	then confused or fabulous
de la navigation.	of navigation.
Entièrement absorbé	Entirely absorbed
dans sa félicité domestique	in his domestic happiness
et dans ses contemplations géographiques,	and in his geographical studies,
il eut un premier fils	he had a first son
qu'il appela Diego,	whom he called Diego,
du nom de son frère.	after the name of his brother.
Sa société intime	His intimate friends
ne se composait que	were composed only
de marins revenant	of sailors returning
des expéditions lointaines,	from distant expeditions,
ou rêvant	or dreaming
des terres inconnues	of unknown lands
et des routes non frayés	and of routes not (yet) traced out
sur l'Océan.	across the Ocean.
Son atelier de cartes et de globes	His workshop for charts and globes
était un foyer d'idées,	was a focus of ideas,
de conjectures, de projets,	conjectures, and projects
qui entretenait sans cesse	which continually occupied
son imagination	his imagination
de quelque grand inconnu	with something great unknown
sur le globe.	on the earth.
Sa femme, fille et sœur de marins,	His wife, the daughter and sister of sailors,
partageait elle-même	herself shared
ces enthousiasmes.	this enthusiasm.
En contournant	In turning round
sous ses doigts ses globes,	his globes with his fingers
et en pointant ses cartes	and in studding his maps
d'îles et de continents,	with islands and continents,
un vide immense	a vast blank (vacuum)
avait frappé les yeux de Colomb	had struck the eyes of Columbus
au milieu de l'Océan Atlantique,	in the midst of the Atlantic.
La terre semblait manquer,	The world seemed to lack
là, du contre-poids	there, the counter-balancing weight
d'un continent.	of a continent.
Des rumeurs vagues,	Rumours indistinct,
merveilleuses, terribles,	marvellous, terrible,
parlaient à l'imagination	appealed to the imagination
des navigateurs de côtes	of the navigators of coasts
entrevues du sommet des Açores,	seen from the summit of the Azores,
dites mobiles ou flottantes	called moving or floating
qui se montraient	which showed themselves
par des temps sereins,	in calm weather,
qui disparaissaient	which disappeared
ou qui s'éloignaient	or which went away
quand les pilotes téméraires	when the rash pilots
cherchaient à en approcher.	sought to approach them.
Un voyageur vénitien,	A Venetian traveller,
Marco Polo,	Marco Polo,
qu'on regardait alors	whom people looked upon
comme un inventeur de fables	then as an inventor of fables
et dont le temps	and whose truthfulness time
a reconnu depuis la véracité,	has since acknowledged,
racontait à l'Occident	narrated to the West,
les merveilles des continents,	the wonders of the continents,

des États et des civilizations
de la Tartarie, de l'Inde,
de la Chine,
que l'on supposait se prolonger
là où s'étendent en réalité
les deux Amériques.
Colomb lui-même
se flattait de trouver,
à l'extrémité de l'Atlantique,
ces contrées de l'or
des perles et de la myrrhe,
dont Salomon
tirait ses richesses ;
cet Ophir de la Bible,
recouvert depuis des nuages
du lointain et du merveilleux.

Ce n'était pas
un continent nouveau,
mais un continent perdu
qu'il cherchait.
L'attrait du faux
le menait à la vérité.
Il supposait
dans ces calculs,
d'après Ptolemée et
d'après les géographes arabes,
que la terre était un globe
dont on pourrait
faire le tour.
Il croyait ce globe
moins vaste qu'il ne l'est
de quelques milliers de lieues.

Il s'imaginait,
en conséquence,
que l'étendue de mer
à parcourir pour arriver
à ces terres inconnues
de l'Inde
était moins immense
que les navigateurs
ne le pensaient.
L'existence de ces terres
lui semblait confirmée

par les témoignages étranges
des pilotes
qui s'étaient avancés
le plus loin
au delà des Açores.
Les uns avaient vu flotter
sur les vagues
des branches d'arbres
inconnus en Occident ;
les autres,
des morceaux de bois sculptés,
mais qui n'avaient pas été
travaillés à l'aide
d'outils de fer ;
ceux-là,
des sapins monstrueux
creusés en canot
d'un seul tronc,
qui pouvaient porter
quatre-vingt rameurs ;
ceux-ci,
des roseaux gigantesques ;
d'autres, enfin,
des cadavres d'hommes
blancs ou cuivrés,
dont les traits

of the States and civilizations
of Tartary, of India,
of China,
which people thought stretched
there where in truth extended
the two Americas.
Columbus himself
cherished the idea of discovering
at the extremity of the Atlantic,
those countries of gold
pearls and myrrh,
from-which Solomon
drew his riches ;
that Ophir of the Bible
hidden since in the clouds
of the distant and wonderful (past).

It was not
a new (continent),
but a lost continent
that he sought.
The attraction of the false
led him to the truth.
He supposed
in his calculations,
according to Ptolemy and
the Arabian geographers
that the earth was a sphere
of which one could
make the tour.
He believed this globe (to be)
less vast than it is
by some thousands of leagues.

He imagined,
in consequence (therefore)
that the extent of sea
to traverse in order to arrive
at those unknown countries
of India
was less immense
than the navigators
thought it (to be).
The existence of these lands
seemed to him (to be) confirmed

by the strange testimony
of the pilots
who had advanced
the farthest
beyond the Azores.
Some had seen floating
on the waves
branches of trees
unknown in the West ;
others,
pieces of wood carved,

but which had not been
worked by the help
of iron tools ;
others,
monster pine trees
hollowed out like canoes
from a single trunk,
which could carry
80 rowers ;
some (had seen floating)
gigantic reeds ;
others, finally,
corpses of men
white or copper-coloured,
whose features

ne rappelaient en rien
les races occidentales,
asiatiques ou africaines.
Tous ces indices
flottants de temps en temps
à la suite des tempêtes
sur l'Océan,
et je ne sais
quel instinct vague
qui précède toujours
les réalités
comme l'ombre
précède le corps
quand on a le soleil
derrière soi,
annonçaient au vulgaire
des merveilles,
attestaient à Colomb
des terres existantes
au delà des plages écrites
par la main des géographes
sur les mappemondes.
Seulement il était convaincu
que ces terres
n'étaient qu'un prolongement
de l'Asie, remplissant
plus d'un tiers
de la circonférence du globe.

Cette circonférence,
ignorée alors
des philosophes et des géomètres,
laissait aux conjectures
l'étendue de cet Océan
qu'il fallait traverser

pour atteindre
à cette Asie imaginaire.
Les uns la croyaient
incommensurable ;
les autres se la figuraient

comme une espèce
d'éther profond et sans borne,
dans lequel
les navigateurs s'égarent,
comme aujourd'hui
les aéronautes
dans les déserts
du firmament.
Le plus grand nombre,
ignorant les lois
de la pesanteur
et de l'attraction
qui rappelle les corps au centre,
et admettant néanmoins déjà

la rotondité du globe,
croyaient que des navires
ou des hommes
portés par le hasard
aux antipodes
s'en détacheraient

pour tomber
dans les abîmes de l'espace.
Les lois qui gouvernent
les niveaux et les mouvements
de l'Océan
leur étaient également inconnues.
Ils se représentaient la mer,

au delà d'un certain horizon
borné par les îles

in no way recalled
the Western, Asiatic,
or African races.
All these indications
floating from time to time
in the wake of storms
on the Ocean,
and I know not
what mysterious instinct
which always precedes
realities
as the shadow
precedes the body
when one has the sun
behind oneself,
announced to the people
wonders,
proved to Columbus
countries existing
beyond the shores described
by the hand of geographers
on the map of the world.
Only he was convinced
that these lands
were merely a prolongation
of Asia, occupying
more than one third
of the circumference of the globe.

This circumference,
unknown until then
by philosophers and geometricians
left to conjecture
the extent of this Ocean
which it was necessary to cross

in order to arrive
at this imaginary Asia.
Some thought it
incommensurable ;
others pictured it to themselves

as a kind
of ether deep and illimitable,
in which
navigators get lost,
just as at the present time
aeronauts (are lost)
in the vast-waste
of the firmament.
The greatest number
ignorant of the laws
of gravity.
and (of) attraction
which attracts bodies to the centre,
and admitting nevertheless already

the roundness of the globe,
thought that vessels
or men
carried by chance
to the antipodes
would become detached from them

so as to fall
into the abyss of space.
The laws which govern
the levels and the movements
of the Ocean
were equally unknown to them.
They pictured to themselves the sea

beyond a certain horizon
bounded by the islands

déjà découvertes, already discovered,
comme une sorte as a kind
de chaos liquide, of watery chaos,
dont les vagues démesurés whose vast billows
s'élevaient reared themselves
en montagnes inaccessibles, like (in) inaccessible mountains,
se creusaient sank down
en gouffres sans fond, like (in) bottomless gulfs,
se précipitaient du ciel rushed from the heavens
en cataractes infranchissables like (in) insurmountable cataracts
qui entraîneraient which would carry away
et engloutiraient and engulf
les voiles assez téméraires those vessels rash enough
pour en approcher. to approach them.
Les plus instruits, The most learned,
en admettant les lois while acknowledging the laws
de la pesanteur of gravity
et un certain niveau and a certain level
dans les espaces liquides, in the watery wastes,
pensaient thought
que la forme arrondie du globe that the round shape of the globe
donnait à l'Océan gave to the Ocean
une pente vers les Antipodes, a slope towards the Antipodes,
qui emporterait les vaisseaux which would carry off the vessels
vers les rivages sans nom, towards nameless shores,
mais qui but which
ne leur permettrait jamais would never allow them
de remonter cette pente to remount this slope
pour revenir en Europe. to return to Europe.
De ces préjugés divers From these various prejudices
sur la nature, la forme, on the nature, shape,
l'étendue, les montées extent, ascent (height)
et les descentes de l'Océan, and descent of the Ocean,
se composait there arose
une terreur générale et mystérieuse a universal and mysterious terror
qu'un génie investigateur which an investigating genius
pouvait seul aborder alone could handle
par la pensée, in thought,
et qu'une audace surhumaine and which a superhuman audacity
pouvait seule affronter could alone assault
de ses voiles. with its ships.
C'était la lutte This was the struggle
de l'esprit humain of the human mind
contre un élément ; against an element ;
pour la tenter, to tempt this (element)
il fallait plus qu'un homme. demanded more than man.

CHAPTER IX.

L'attrait invincible The unconquerable attraction
du pauvre géographe, of the poor geographer
vers cette entreprise towards this enterprise
était le véritable lien was the real bond (tie)
qui retenait tant d'années which detained for so many years
Colomb à Lisbonne Columbus at Lisbon
comme dans la patrie as in the native country
de ses pensées. of his thoughts.
C'était le moment It was the moment
où le Portugal, when Portugal
gouverné par Jean deux, governed by John the Second
prince éclairé et entreprenant, an enlightened and enterprising prince,
se livrait, dans un esprit gave itself up, in a spirit
de colonisation, of colonization,
de commerce et d'aventures, commerce and adventure,
à des tentatives navales incessantes to incessant naval attempts
pour relier l'Europe to bind Europe
à l'Asie, to Asia,
et où Vasco de Gama, and when Vasco de Gama,
le colon portugais, the Portuguese colonist,
n'était pas loin de découvrir was not far from discovering
la route maritime des Indes the maritime route to India
par le cap by the Cape
de Bonne-Espérance. of Good Hope.
Colomb, convaincu Columbus, convinced
qu'il trouverait that he would find
une route plus large et plus directe a larger and more direct route
en s'élançant by starting (setting out)
droit devant lui straight before him
vers l'ouest, obtint, towards the west, obtained,
après de longues sollicitations, after long solicitation,
une audience du roi, an audience of the king,
pour lui révéler in order to disclose to him
ses plans de découverte his plans of discovery
et pour lui demander and to ask of him
les moyens de les accomplir the means of accomplishing them
au profit de la fortune for the benefit of the fortune
et de la gloire and the glory
de ses États. of his States.
Le roi l'écouta avec intérêt. The King listened to him with attention.
La foi de cet inconnu The faith of this unknown man
dans ses espérances in his hopes
ne lui parut pas did not seem to him
assez dénuée de fondement sufficiently devoid of foundation
pour la reléguer so as to relegate it
au rang des chimères. to the rank of dreams.
Colomb, indépendamment Columbus, independently
de son éloquence naturelle, of his natural eloquence
avait l'éloquence had the eloquence
de sa conviction. of his convictions.
Il émut assez le roi He moved the King so far
pour que ce prince chargeât that he ordered
un conseil composé de savants a Council composed of savants
et de politiques and of politicians
d'examiner les propositions to examine the proposals
du navigateur génois, of the Genoese navigator,
et de lui faire un rapport and to give him a report
sur les probabilités on the probabilities
de son entreprise. of his enterprise.
Ce conseil, composé This Council, composed
du confesseur du roi of the King's confessor
et de quelques géographes and of some geographers
d'autant plus accrédités (by) so much the more influential
dans sa cour at his court
qu'ils s'écartaient moins the less they were removed
des préjugés vulgaires, from the prejudices of the vulgar,
déclara les idées de Colomb declared the ideas of Columbus
chimériques et contraires chimerical and contrary
à toutes les lois to all the laws
de la physique et de la religion. of nature and religion.
Un second conseil d'examen, A second Council of examination
auquel Colomb en appela to which Columbus appealed
avec la permission du roi with the permission of the King
aggrava encore aggravated (confirmed) still more
cette première décision. this first decision.
Toutefois, par une perfidie However, through treachery
ignorée du roi, unknown to the King,

ses conseillers communiquèrent	his councillors communicated
les plans de Colomb a un pilote,	the plans of Columbus to a pilot,
et firent partir secrètement	and despatched secretly
un navire pour tenter	a vessel to try,
à son insu	unknown to him,
là route qu'il indiquait	the route which he had pointed out
vers l'Asie.	towards Asia.
Ce navire, qui avait cinglé	This vessel which had sailed
quelques jours	some days
au delà des îles Açores,	beyond the island of the Azores,
revint épouvanté du vide	returned terrified at the void waste
et de l'immensité de l'espace	and the vastness of the space
qu'il avait entrevue,	which it had seen,
et confirma le conseil	and confirmed the Council
dans le mépris	in their contempt
des conjectures de Colomb.	for the theories (fancies) of Columbus.

Notes.

CHAPTER VIII.

Y (there) is most frequently translated by a preposition and a pronoun = in it, in them, &c. Espérer sometimes takes *de* + infin. Continuer à + infin. *Par* un soleil brûlant. *Par* des temps sereins. Par = during. Approcher (see Dictionary). Supposer (no prep.) Flatter de + infin. Dont = from which. Ne must be used with the verb after *plus*, *mieux*, *moins*, *meilleur*, *autrement* when used affirmatively.

Translate into French: The soldiers escaped from the flames. The sailor seized an oar with one hand and swam with the other to the shore. All Spain was carried away (saisir) with a passion for maritime discoveries. The boy was called John. The geographer continued making charts. He was called Diego after his father's name. He incessantly fed his imagination with these noble thoughts. The master turned the globe round with his fingers. The globe wanted one continent. He made me come near him. Come near the fire. The ships which he was approaching were very large. They supposed these continents to be prolonged. He flattered himself with having found the countries from which Solomon drew his riches. This globe is smaller than you suppose by some thousands of leagues. The pilots had advanced beyond these islands. He saw the corpse floating on the billows.

En = like. En ami = like a friend. Savoir, oser, pouvoir often omit the negative *pas*. Plus takes de (= than) before numerals. Falloir omits preposition before infin. En = from it. Permettre de + infin.

Translate into French: This circumstance was ignored by the philosopher. It is necessary to cross this ocean in order to reach Asia. Some pictured it to themselves as an ocean without bounds. Picture it to yourself. The navigators will become lost. Detach yourself from it. I am not rash enough to approach it (or them). Permit me to offer you a glass of wine. Superhuman audacity alone could brave these dangers with its vessels.

Grammatical Questions.

CHAPTER VIII.

1. Give the homonyms of 'sens,' with their meaning.

2. Give the plural and the masculine form of belle, mère.

3. Account for the 'e' in partageaient. Write the past definite of s'élancer and present indicative and future of appela, espérer.

4. 'Le Portugal.' Of what gender are the names of countries, give exceptions.

5. Give the respective derivations of lieu, lieue, droit, savoir, jour, car.

6. On what principles are the rules about the gender of substantives based? Explain the gender of passage, service, obscurité, papier, lois.

7. 'Un vide immense,' why not 'un immense vide.' What other adjectives are placed after the noun?

8. 'Les uns avaient vu . . . africaines.' Account for the agreement of the past participles.

9. 'Quatre-vingts rameurs.' When do 'vingt' and 'cent' take the sign of the plural? Translate: 100 soldiers, 305 leagues, 800 men.

10. Parse 's'étaient avancés, flottant, la and là, dites in: du sommet des Arçons, dites mabiles.

11. Is it right that the present participle in 'Tous ces indices flottants...l'Ocean' should agree with the noun indices? Translate: He speaks without ceasing, annoying everyone with the tales of his adventures. I know certain works the reading of which is fatiguing. When does the present participle agree with its noun, and when not?

12. Explain the genders of nombre, espace, fable, nom, globe, corps, ésprit.

13. Explain the so-called irregular forms: Je reçoive, je doive, je veuille, j'aille.

14. La voile, le voile—le mousse, la mousse, le manche la manche, le, la souris.

(To be continued.)

The Musical Instrument (Violin) Maker of Cremona.

Scène IV.	Scene IV.
Les Mêmes, Maître Ferrari.	The Same, Master Ferrari.
Master Ferrari.	
[*un peu plus gris que d'abord, et portant un panier à bouteilles.*	[*a little more intoxicated than before, and carrying a wine basket.*
C'est étrange !	It is strange !
Voilà plus de vingt ans, mes amis, que je range	For more than twenty years, my friends, I have arranged
Mes deux sortes de vin en un lieu bien couvert :	My two sorts of wine in a well covered place :
A droite, cachet rouge : à gauche, cachet vert.	To the right, red seal : to the left, green-seal.
Personne n'entre là, j'ai la clef dans ma poche.	Nobody enters there, I have the key in my pocket.
Eh bien, je viens de voir, soit dit sans nul reproche,	Well, I just saw, be it said without any reproach
Que mes vins, qui sont là de toute éternité,	That my wines, which have been there from all eternity,
M'ont fait le mauvais tour de changer de côté.	Have played me the bad trick to change sides.
Hein ! Mes flacons entre eux font-ils donc la débauche,	Hey ! Are my flagons giving way to a fit of unruliness,
Ou ne connais-je plus ma droite de ma gauche ?	Or do I know no longer my right from my left ?
Giannina.	
Mon père.....	Father.....
Master Ferrari.	
Toi, fillette...Eh bien, je te cherchais.	You, daughter...Well, I was looking for you.
Tantôt, quand on aura fait grincer les archets	As soon as they have scraped on the fiddles
Et qu'enfin nous saurons à qui je te marie,	And we at last know to whom I shall marry you,
Je reçois à dîner toute la confrérie.	I shall receive (invite) the whole confraternity to dinner.'

Viens donc m'aider à mettre, afin que je sois beau,
Ma perruque de fête et mon habit barbeau.
En manquant de tenue, un homme se dégrade.
Viens.

[*Il sort à droite, suivi de Giannina.*

Come and help me to put on, so that I may be smart,
My best wig and my light-blue coat.
When neglecting his personal appearance a man lowers himself.
Come.

[*Exit to the right, followed by Giannina.*

Scène V.

Filippo, Sandro.

Scene V.

Filippo, Sandro.

Sandro.

L'instant décisif approche.

The decisive moment approaches.

Filippo.

Oui, camarade.

Yes, comrade.

Sandro.

Ton violon est prêt ?

Is your violin ready ?

Filippo.

Il l'est.

It is.

Sandro

Es-tu content ?

Are you satisfied ?

Filippo.

Oui, bien sincèrement. Et toi ?

Yes, very much. And you ?

Sandro.

Moi ? Pas autant.

I ? Not so much.

Filippo.

Tant pis. Dans la courtoise et fraternelle lutte,
Ce qui m'aurait le mieux consolé d'une chûte
C'eût été ton succès, à toi, mon compagnon.
Voyons, Sandro, veux-tu me donner la main ?

So much the worse. In the courteous and fraternal contest,
What would have most consoled me in defeat
Would have been your success, my companion.
Come, Sandro, will you shake hands with me ?

Sandro.

[*après un silence.*
Non !
[*Il sort brusquement.*

[*after some silence.*
No !
[*Exit abruptly.*

Scène VI.

Scene VI.

Filippo.

[*seul.*

Un envieux ! Voilà que le chagrin commence !
Il souffre. Pardonnons. Mais, parbleu ! c'est démence
De croire qu'à l'ami pauvre et deshérité
Qui n'envia jamais sa force et sa beauté,
Un homme trouve un jour le plus mince mérite
Sans que son amour-propre aussitôt s'en irrite.
Ce serait bien pourtant : être amis et rivaux.
Celui-là ne sait pas non plus ce que tu vaux

[*alone.*

An envious man ! Now the trouble begins !
He suffers. Let us pardon him. But, by Jove ! it is madness
To believe that in a poor and disinherited friend
Who never envied his strength and beauty,
A man should find some day the smallest merit
Without his self-love being immediately irritated.
That would really mean : to be friends and rivals.
This one does not know what thou art worth

O cœur plein de tendresse et que le monde isole !
Mais mon chet d'œuvre est là qui de tout me console.
Pauvre cher violon ! Je suis pareil à lui,
Instrument délicat dans un informe étui.

[*Il va prendre, dans une armoire, son violon qui est renfermé dans un étui rouge, et le pose sur la table à droite.*

O heart full of tenderness, and which the world isolates !
But my master-piece is here which consoles me for everything.
Poor dear violin ! I am like it,
A delicate instrument in a shapeless case.

[*He takes from a cupboard his violin which is enclosed in a red case, and places it on the table on the right.*

Viens, je veux te revoir encore, ô mon ouvrage.
Chère création sur qui j'eus le courage
Moi, l'ouvrier débile et dévoré d'ennuis,
De passer au travail tant de jours et de nuits !
Viens, de ton sein profond va jaillir tout à l'heure
Le scherzo qui babille et le lento qui pleure.
Sur le monde, tu vas répandre, ô mon ami,
Le sublime concert dans ton sein endormi.
Viens, je veux te revoir et te toucher encore !
Je n'éveillerai pas ton haleine sonore,
Mais je veux seulement voir mon regard miré
Une dernière fois dans ton beau bois doré.
Car il faut nous quitter pour ta gloire et la mienne ;
Mais, dans ta vie, ami, noble ou bohémienne,
Que tu fasses danser le peuple des faubourgs,
Ou que devant les grands du monde et dans les cours,
Tu frémisses aux doigts des puissants virtuoses,
Moi, qui, naïvement, crois à l'esprit des choses,
En te disant adieu, je viens te supplier,
Noble et cher instrument, de ne pas oublier
Celui qui t'a donné tes beaux accents de flamme
Et le pauvre bossu qui t'a soufflé son âme !

[*Il remet le violon dans son étui.*

Come, I must see thee once again, O my work.
Dear creation, over which I had the courage
I, the workman, ailing and tired out with weariness,
To pass working so many days and nights !
Come, from the depth of thy heart will presently burst forth
The scherzo which prattles and the lento which wails.
Over the world thou wilt pour forth, oh my friend,
The sublime concert slumbering in thy bosom.
Come, I must see and touch thee once more !
I shall not awake thy sonorous breath,
But I only want to see my look reflected
For the last time in thy beautiful gilded wood.
For we must part for thy glory and my own ;
But in thy life, friend, be it noble or plebeian,
Whether thou makest the people of the suburbs dance,
Or that before the great in the world and in the courts
Thou shouldst tremble under the fingers of mighty virtuosos,
I, who, in all simplicity, believe in the spirituality of things,
In saying good-bye, I wish to entreat thee,
Noble and dear instrument, not to forget
Him who has given thee thy beautiful and fiery notes
And the poor hunchback who has breathed his very soul into thee !

[*He puts back the violin into the case.*

Mais quel enfant je suis ! Et puis, non, je me mens
A moi-même et j'étouffe en vain mes sentiments.
Pauvre fou ! Ce n'est pas seulement pour la gloire
Que j'ai voulu sur tous gagner cette victoire

But what a child am I ! And then, I belie
Myself, and I repress in vain my sentiments.
Poor fool ! It is not only for glory
That I wished to gain this victory over all

Et que sur ce travail mon effort s'obstina ;
C'est pour elle, la douce et belle Giannina.
Car seule elle eut pitié de ma peine en ce monde,
Et lorsque mon enfance errante et vagabonde
S'arrêta sur le seuil de maître Ferrari,
Si bonne en m'accueillant, seule, elle n'a pas ri.
Non, ce muet amour de son ami d'enfance,
Giannina ne peut pas le prendre pour offense,
Ni m'en vouloir du grand désir que j'ai formé
D'être assez glorieux un jour pour être aimé.
Oh ! certes, si j'obtiens le succès que j'espère,
Je ne m'armerai pas du serment de son père.....
Mais peut-être...qui sait ?... son cœur est libre encore...
Quand je lui donnerai la belle chaîne d'or,
Quand elle sentira que de ce corps si frêle
La flamme du génie a pu jaillir pour elle,
Elle est fille d'artiste, elle aura la grandeur,
En songeant au talent, d'oublier la laideur,
Et par tant de raisons son âme combattue
Pourrait bien.....Oh ! je fais un rêve qui me tue !

And that to this work all my efforts were directed ;
It is for her, the sweet and beautiful Giannina.
For she alone had compassion with my trouble in this world,
And when an abandoned and wandering child
I stopped on Master Ferrari's threshold,
So kindly she received me, and she alone did not laugh.
No, this mute love of the friend of her childhood,
Giannina cannot take as an offence,
Nor be angry with me on account of the great desire I have formed
To become some day famous enough to be worth loving.
Oh ! certainly, if I obtain the success for which I hope,
Then I shall not arm myself with her father's promise...
But perhaps...who knows... her heart is yet free...
When I shall give her the pretty golden chain,
When she will see that from this frail body
The flame of genius could burst forth for her sake,
She is the daughter of an artist, she will be high-minded enough,
In considering my talent, to forget my deformity.
And her soul subdued by so many reasons
Might well.....Oh ! I dream a dream that kills me !

Scène VII.

Filippo, Giannina.

Scene VII.

Filippo, Giannina.

Giannina.

[*entrant.*

Il est seul...Ah ! Je veux lui parler et savoir
Si Sandro peut encore conserver quelque espoir.

[*entering.*

He is alone...Ah ! I must speak to him to know
If Sandro may yet nourish any hope.

[*haut.*

Filippo !

[*aloud.*

Filippo !

Filippo.

[*sortant de sa rêverie.*

Dieu, c'est elle !

[*awaking from his day-dream.*

Heavens, it is she !

Giannina.

Il faut que je vous gronde,
Car seule j'ignorais ce que sait tout le monde,
Et ce n'est pas par vous que je l'ai même appris.

I have to scold you,
For I alone knew not what all the world knows,
And it is not even through you that I have heard of it.

Filippo.

Quoi donc ?

What then is it ?

Giannina.

Que vous allez concourir pour le prix.

That you are going to compete for the prize.

Filippo.

A vous il eût fallu d'abord que je l'apprisse,
C'est vrai. Mais quand j'ai su le singulier caprice
Du maître et le serment qu'il avait prononcé,
Pardon, signorina, mais je n'ai plus osé.

I should have told you first about it,
It is true. But when I got to know of the strange fancy
Of the master and the promise he had made,
Pardon me, miss, but then I no longer dared.

Giannina.

Oui, mais laissons cela. Mon vieux père qui m'aime
Ne voudra pas ainsi disposer de moi-même
Ni charger le hasard du soin de mon bonheur.
Quant à la chaîne d'or, quant au brevet d'honneur,
C'est différent, chacun a le droit d'y prétendre,
Et vous surtout, d'après ce que je viens d'apprendre.

Yes, but let us leave that. My aged father who loves me
Will never thus dispose of me
Nor entrust to chance the care of my happiness.
With the golden chain and the place of honour,
It is different, everybody has a right to compete for them
And you above all, after what I have just learnt.

Filippo.

Comment ?

What do you mean ?

Giannina.

Mais vous avez construit un instrument
Qui, dit-on, doit avoir le prix, certainement.
Un chef d'œuvre.....

But you have made an instrument
Which, they say, must win the prize for a certainty.
A master-piece.....

Filippo.

J'ai fait de mon mieux, je l'avoue ;
Mais que dans le concours je triomphe ou j'échoue,
Qui donc, signorina, s'en préoccuperait ?

I have done my best, I confess ;
But whether I am successful in the competition or whether I fail,
Who then, Miss, would trouble about it ?

Giannina.

Qui ? mais bien des amis vous portent intérêt,
Vous l'ont même prouvé.

Who ? But a good many friends are interested in you,
Have even proved it to you.

Filippo.

Pardon, je suis stupide.
On paraît méfiant quand on n'est que timide,
Et de tous mes secrets je vous dois la moitié.
Quand j'avais du chagrin, vous avez eu pitié ;
Vous vous réjouirez d'un bonheur, s'il m'arrive.
Mais je suis devenu comme la sensitive,
Et quand on vient vers moi, d'un geste machinal
Je recule et je crois qu'on veut me faire mal.
Pardonnez !

Pardon me, I am foolish.
One appears distrustful whilst one is merely timid,
And of all my secrets I owe one half to you.
When I was in trouble, you have shown sympathy ;
You will rejoice in my good fortune if it comes.
But I have become like the sensitive plant,
And if they approach me with a mechanical movement
I fall back and think they want to hurt me.
Pardon me !

Giannina.

S'il en est ainsi, je me retire.

If it is thus, I will retire.

Filippo.

Non, par grâce, restez ! Non, je veux tout vous dire,
Car j'étais un ingrat et je vous offensais.
Sachez-le donc, je suis presque sûr du succès.
Je juge mon travail sans aucune indulgence,
Est-ce talent ou bien seulement bonne chance ?
Je l'ignore, mais j'ai tout à fait réussi.

No, for mercy's sake, stay ! No, I will tell you all,
For I was ungrateful and offended you.
Know then, I am almost certain of success.
I judge my work without any prejudice,
Is it talent or only good luck ?
I do not know, but I have succeeded completely.

[*Montrant son violon.*

Lorsque j'ai commencé mon œuvre que voici,
J'ai bien construit, avec tout le soin désirable,
Lá boîte en vieux sapin et le manche en érable,
Bref, j'ai fait de mon mieux ...mais cela n'était rien ;
Les autres violons peuvent être aussi bien.....
Non, voyez-vous, mon coup de maître, ma merveille,
C'est que j'ai retrouvé, dans une heure de veille,
Le vernis d'autrefois, le secret envolé.....

[*Showing his violin.*

When I began this work of mine which you see,
I have constructed, with all desirable carefulness,
The box of old fir-wood and the handle of maple-wood,
In short, I have done my best.....but that is all nothing ;
The other violins may be as good.....
No, you see, my master-stroke, my marvel,
Is that I discovered again, one wakeful night
The varnish of old, the lost secret.....

Giannina.

Quoi ! le fameux vernis des vieux maîtres ?.....

What ! The famous varnish of the old masters ? .....

Filippo.

[*s'animant.*

Je l'ai,
Et je veux dès demain en généreux émule,
A tous mes concurrents en donner la formule.
Allez ! j'ai comparé mon œuvre d'apprenti
Avec un violon de l'illustre Amati :
C'était le même son. Cela tient du prodige !
Juste le même son, oui, le même, vous dis-je !
J'en suis sûr, et, pareil aux maîtres d'autrefois,
Je fais jaillir de ces quatre planches de bois
Une note profonde, immense, magistrale
Et sonore à remplir toute une cathédrale.

I have it,
I shall to-morrow as a generous rival
Give the description of it to all my fellow-competitors.
Let me tell you ! I have compared my work while apprentice
With a violin of the illustrious Amati ;
It was the same tone. It is like a miracle !
Exactly the same tone, yes, the same, I tell you !
I am sure of it, and, like the masters of old,
I draw forth from these four wooden boards
A note, deep, full, masterly
And sonorous enough to fill a whole cathedral.

Giannina.

[*à part.*

Hélas ! pauvre Sandro !

[*aside.*

Alas ! poor Sandro !

Filippo.

Depuis ce jour heureux,
Je cache mon bonheur ainsi qu'un amoureux.
Que m'importe à présent ? ma vie est une fête.
Je jouis, en avare et seul, de mon trésor.
Tous les matins, avant qu'il fasse jour encor,

Since this blessed day,
I hide my happiness as a lover does.
What does it matter to me now? My life is one (continual) fete.
I rejoice like a miser and alone in my treasure.
Every morning, before it is yet day-light,

Je traverse Crémone endormie et je gagne
Un endroit que je sais, là-bas, dans la campagne,
Avec mon violon caché sous mon manteau.
Là, je m'assieds, tout seul, au versant d'un côteau,
Dans le gazon trempé de rosée, et je rêve
Jusqu' à l'heure sublime où le soleil se lève.
Enfin, quand l'horizon s'emplit de diamants,
Lorsque s'annonce, avec de longs frémissements,
Autour de moi le grand réveil de la nature,
Lorsque l'herbe frissonne et que le bois murmure,
Et que des buissons verts par la nuit rajeunis
S'échappe le concert éblouissant des nids,
Je prends mon violon, joyeux, et j'improvise !
Ah ! voyez-vous, c'est là la récompense exquise ;
Et j'accompagne alors d'un archet triomphant
Tous les bruits glorieux dans le soleil levant,
Ces longs soupirs du vent à travers la feuillée
Et ces gazouillements de volière éveillée.
Je joue avec ivresse, et l'instrument vainqueur
Que je sens tressaillir là, tout près de mon cœur,
Mêle à ses chants d'aurore où mon âme se noie
Un hymne merveilleux de jeunesse et de joie !

I traverse sleeping Cremona and I reach
A place which I know down there in the country,
With my violin hidden beneath my cloak.
There, I sit down, quite alone, on the side of a hill,
On the turf, moistened by the dew and I muse
Until the sublime hour when the sun rises.
At last, when the horizon seems to be sparkling with diamonds,
When with a protracted quivering there announces itself,
Around me the great awakening of nature,
When the grass trembles and a murmuring is heard in the wood,
And when from the green bushes, refreshed by the night,
Issues the dazzling concert of the nests,
I take my violin joyfully and I improvise !
Ah ! You see, this is an exquisite recompense ;
And I then accompany with triumphant bow
All these glorious sounds whilst the sun is rising,
Those long sighs of the wind through the foliage
And those warblings of the awakened aviary.
I play with exultation, and the victorious instrument
Which I feel vibrating here quite near to my heart,
Mingles with these morning-songs wherein my soul loses itself,
A marvellous song of youth and joy !

Giannina.

[*à part.*

Pauvre, pauvre Sandro !

[*aside.*

Poor, poor Sandro !

[*haut.*

C'est si beau que cela !

[*aloud.*

That is so good !

Filippo.

[*mettant son violon à l'épaule.*

Écoutez seulement comme il donne le 'la.'

[*raising his violin to his shoulder.*

Listen only how it gives the 'la.'

Giannina.

Oh ! jouez donc un air ! Je voudrais mieux l'entendre.

Oh ! play an air ! I should like better to hear it.

Filippo.

[*à part.*

En me priant ainsi, sa voix est presque tendre.
Mon Dieu ! pour mon succès ferait-elle des vœux ?

[*aside.*

In asking me, her voice is almost tender.
Good heavens ! Can it be that she prays for my success.

[*haut.*

Vraiment, signorina, vous voulez ?

[*aloud.*

Really, Miss, you insist ?

(To be continued.)

Query Column.

RULES.

1. Each correspondent is restricted to *one question*. We should be much obliged if correspondents, who send questions for solution, would give (if possible) the required answer, and the source from which the question is obtained.

2. No query can be answered unless accompanied by the real name and address of the sender, not necessarily for publication, but as a guarantee of good faith and for facility of reference.

3. ☞ **When a pseudonym is adopted it should be written at the end of the query, and the real name and address on a separate piece of paper.**

4. Correspondents are requested to write their queries *legibly*, and on one side of the paper only.

5. Replies will not be sent through the post.

6. Queries must reach the office *not later than the 12th of the month*, or they cannot be attended to in the following issue.

*** All communications for this column should be addressed—'*The Query Editor*,' *The Practical Teacher*, *Pilgrim Street, Ludgate Hill, London, E.C.*

Algebra.

1. FRANK SPENCE.—Given

$$l_1^2 + m_1^2 + n_1^2 = 1 \ldots\ldots i.$$
$$l_2^2 + m_2^2 + n_2^2 = 1 \ldots\ldots ii.$$
$$l_3^2 + m_3^2 + n_3^2 = 1 \ldots\ldots iii.$$
$$\text{and } l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \ldots\ldots iv.$$
$$l_2 l_3 + m_2 m_3 + n_2 n_3 = 0 \ldots\ldots v.$$
$$l_3 l_1 + m_3 m_1 + n_3 n_1 = 0 \ldots\ldots vi.$$

Prove $l_1^2 + l_2^2 + l_3^2 = 1 \ldots\ldots a$

$$m_1^2 + m_2^2 + m_3^2 = 1 \ldots\ldots b$$
$$n_1^2 + n_2^2 + n_3^2 = 1 \ldots\ldots c$$
$$\text{and } l_1 m_1 + l_2 m_2 + l_3 m_3 = 0 \ldots\ldots d$$
$$m_1 n_1 + m_2 n_2 + m_3 n_3 = 0 \ldots\ldots e$$
$$n_1 l_1 + n_2 l_2 + n_3 l_3 = 0 \ldots\ldots f$$

and that the determinant

$$\begin{vmatrix} l_1, & m_1, & n_1 \\ l_2, & m_2, & n_2 \\ l_3, & m_3, & n_3 \end{vmatrix} = \pm 1 \ldots\ldots g$$

iv. $\times n_3$; $l_1 l_2 n_3 + m_1 m_2 n_3 + n_1 n_2 n_3 = 0$

vi. $\times n_2$; $l_3 l_1 n_2 + m_3 m_1 n_2 + n_1 n_2 n_3 = 0$.

Subtracting; $-l_1 (n_2 l_3 - n_3 l_2) + m_1 (m_2 n_3 - m_3 n_2) = 0$

$$\therefore \frac{l_1}{m_2 n_3 - m_3 n_2} = \frac{m_1}{n_2 l_3 - n_3 l_2} = \frac{n_1}{l_2 m_3 - l_3 m_2} \ldots vii.$$

we add the last equality by symmetry.

Each of these fractions, by the laws of proportion, are equal to

$$\sqrt{\left\{\frac{l_1^2 + m_1^2 + n_1^2}{(m_2 n_3 - m_3 n_2)^2 + (n_2 l_3 - n_3 l_2)^2 + (l_2 m_3 - l_3 m_2)^2}\right\}}$$

$$= \sqrt{\left[\frac{l_1^2 + m_1^2 + n_1^2}{(l_2^2 + m_2^2 + n_2^2)(l_3^2 + m_3^2 + n_3^2) - (l_2 l_3 + m_2 m_3 + n_2 n_3)^2}\right]}$$

$= \sqrt{1}$ by i., ii., iii., and v.

$$\therefore \frac{l_1}{m_2 n_3 - m_3 n_2} = \frac{m_1}{n_2 l_3 - n_3 l_2} = \frac{n_1}{l_2 m_3 - l_3 m_2} = \pm 1 \ldots viii.$$

$$\begin{vmatrix} l_1, & m_1, & n_1 \\ l_2, & m_2, & n_2 \\ l_3, & m_3, & n_3 \end{vmatrix}$$

$= l_1 (m_2 n_3 - m_3 n_2) + m_1 (n_2 l_3 - n_3 l_2) + n_1 (l_2 m_3 - l_3 m_2)$

$= \pm [(m_2 n_3 - m_3 n_2)^2 + (n_2 l_3 - n_3 l_2)^2 + (l_2 m_3 - l_3 m_2)^2$ by vii. and viii.

$= \pm [(l_2^2 + m_2^2 + n_2^2)(l_3^2 + m_3^2 + n_3^2) - (l_2 l_3 + m_2 m_3 + n_2 n_3)^2$

$= \pm 1$ by ii., iii., and v., which proves g.

Similarly from iv. and v.

$$\frac{l_2}{m_3 n_1 - m_1 n_3} = \frac{m_2}{n_3 l_1 - n_1 l_3} = \frac{n_2}{l_3 m_1 - l_1 m_3} = \pm 1 \ldots ix.$$

and from v. and vi.

$$\frac{l_3}{m_1 n_2 - m_2 n_1} = \frac{m_3}{n_1 l_2 - n_2 l_1} = \frac{n_3}{l_1 m_2 - l_2 m_1} = \pm 1 \ldots x.$$

Since $l_1 = \pm (m_2 n_3 - m_3 n_2)$ from vii.

$\times l_1$; $\therefore l_1^2 = \pm l_1 (m_2 n_3 - m_3 n_2)$

Similarly $l_2^2 = \pm l_2 (m_3 n_1 - m_1 n_3)$ from ix.

and $l_3^2 = \pm l_3 (m_1 n_2 - m_2 n_1)$ from x.

Adding

$$l_1^2 + l_2^2 + l_3^2 = \pm [l_1 (m_2 n_3 - m_3 n_2) + l_2 (m_3 n_1 - m_1 n_3) + l_3 (m_1 n_2 - m_2 n_1)]$$

$$= \begin{vmatrix} l_1, & l_2, & l_3, \\ m_1, & m_2, & m_3, \\ n_1, & n_2, & n_3, \end{vmatrix}$$

$$= \begin{vmatrix} l_1, & m_1, & n_1, \\ l_2, & m_2, & n_2, \\ l_3, & m_3, & n_3, \end{vmatrix}$$

$= \pm 1$ by g.

but since the sum of three squares is always positive, we must take the positive sign.

$$\therefore l_1^2 + l_2^2 + l_3^2 = 1 \ldots\ldots a$$

similarly we could prove that

$$m_1^2 + m_2^2 + m_3^2 = 1 \ldots\ldots b$$
$$\text{and } n_1^2 + n_2^2 + n_3^2 = 1 \ldots\ldots c$$

lastly

Since $l_1 = \pm (m_2 n_3 - m_3 n_2)$ .....from viii.

$\times m_1$; $l_1 m_1 = \pm m_1 (m_2 n_3 - m_3 n_2)$

Similarly $l_2 m_2 = \pm m_2 (m_3 n_1 - m_1 n_3)$ .....from ix.

and $l_3 m_3 = \pm m_3 (m_1 n_2 - m_2 n_1)$ .....from x.

Adding we have

$$l_1 m_1 + l_2 m_2 + l_3 m_3 = \pm \begin{vmatrix} m_1, & m_2, & m_3 \\ m_1, & m_2, & m_3 \\ n_1, & n_2, & n_3 \end{vmatrix}$$

$$= 0 \ldots\ldots d.$$

since the first and second rows are alike.

Similarly we could prove e and f.

2. R. F.—The sum of the first four terms of a Geometrical Progression is 40, and the sum of the first eight terms is 3280; find the Progression. (*Todhunter's* 'Algebra for Beginners.')

$$S = \frac{a(r^n - 1)}{r - 1};$$

$$\therefore 40 = \frac{a(r^4 - 1)}{r - 1} \quad (1)$$

$$3280 = \frac{a(r^8 - 1)}{r - 1} \quad (2)$$

Divide (2) by (1) :—

Then, $82 = r^4 + 1$

$$r^4 = 81$$
$$\therefore r = \pm 3.$$

The plus value is the one required.

$$(1)\ 40 = \frac{a(r^4 - 1)}{r - 1}$$
$$= \frac{a(81 - 1)}{3 - 1}$$
$$= 40a$$
$$\therefore a = 1.$$

$\therefore$ Series = 1, 3, 9, &c.

3. NEMO.—Prove that if

$$\frac{(a-b)(c-d)}{(a-c)(b-d)} = x,$$

then

(i.) $\frac{1}{x}$ is the resulting expression from an interchange of a, d, or b, c;

(ii.) $1-x$ from an interchange of b, d, or a, c;

(iii.) $\frac{x}{x-1}$ from an interchange of c, d, or a, b; but no alteration takes place in the expression

$$\frac{(1-x+x^2)^3}{x^3(1-x)^3}.$$

(*Matriculation*, June, 1887.)

If $x = \frac{(a-b)(c-d)}{(a-c)(b-d)}$,

(i.) $\frac{1}{x} = \frac{1}{\frac{(a-b)(c-d)}{(a-c)(b-d)}}$

$= \frac{(a-c)(b-d)}{(a-b)(c-d)}$

= interchange of b, c;

or, $\frac{1}{x} = \frac{(b-d)(a-c)}{(c-d)(a-b)}$

$= \frac{(d-b)(c-a)}{(d-c)(b-a)}$

= interchange of a, d;

(ii.) $1-x = 1 - \frac{(a-b)(c-d)}{(a-c)(b-d)}$

$= \frac{(a-c)(b-d)-(a-b)(c-d)}{(a-c)(b-d)}$

$= \frac{ab-bc-ad+cd-ac+bc+ad-bd}{(a-c)(b-d)}$

$= \frac{ab+cd-ac-bd}{(a-c)(b-d)}$

$= \frac{a(b-c)-d(b-c)}{(a-c)(b-d)}$

$= \frac{(a-d)(b-c)}{(a-c)(b-d)}$

$= \frac{(a-d)(c-b)}{(a-c)(d-b)}$

= interchange of b, d;

or, $1-x = \frac{(c-b)(a-d)}{(c-a)(b-d)}$

= interchange of a, c;

(iii.) $\frac{x}{x-1} = \frac{\frac{(a-b)(c-d)}{(a-c)(b-d)}}{\frac{(a-b)(c-d)}{(a-c)(c-d)} - 1}$

$= \frac{(a-b)(c-d)}{(a-b)(c-d)-(a-c)(b-d)}$

$= \frac{(a-b)(c-d)}{ac-bc-ad+bd-ab+bc+ad-cd}$

$= \frac{(a-b)(c-d)}{ac+bd-ab-cd}$

$= \frac{(a-b)(c-d)}{a(c-b)-d(c-b)}$

$= \frac{(a-b)(c-d)}{(a-d)(c-b)}$

$= \frac{(a-b)(d-c)}{(a-d)(b-c)}$

= interchange of c, d;

or, $\frac{x}{x-1} = \frac{(b-a)(c-d)}{(b-c)(a-d)}$

= interchange of a, b;

The working of the last expression is very tedious.

4. H. M.—The sum of £700 was divided among four persons, whose shares were in G. P., and the difference between the greatest and least was to the difference between the means as 37 to 12. What were their respective shares? (*Todhunter.*)

Suppose the least share to be a pounds, the next ar pounds, the third ar^2 pounds, and the greatest ar^3 pounds.

Then (1) $a + ar + ar^2 + ar^3 = 700$
(2) $ar^3 - a = \frac{37}{12}(ar^2 - a)$

(2) $a(r^3-1) = \frac{37}{12}ar(r-1)$

$r^3 - 1 = \frac{37r}{12}(r-1)$

$r^2 + r + 1 = \frac{37r}{12}$

$r^2 + r - \frac{37r}{12} = -1$

$r^2 - \frac{25r}{12} = -1$

$r^2 - \frac{25r}{12} + (\frac{25}{24})^2 = \frac{625}{576} - 1$

$= \frac{49}{576}$

$r - \frac{25}{24} = \pm \frac{7}{24}$

$\therefore r = \pm \frac{7}{24} + \frac{25}{24}$

$= \frac{3}{4}$, or $\frac{4}{3}$.

The value $\frac{3}{4}$ is the one required.

(1) $a(1 + r + r^2 + r^3) = 700$

$a(1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64}) = 700$

$a\left(\frac{27+36+48+64}{27}\right) = 700$

$a \times \frac{175}{27} = 700$

$\therefore a = \frac{4}{700} \times \frac{27}{175}$

$= 108.$

$\therefore$ Shares = £108, £144, £192, £256. Ans.

5. WELSHMAN.—Suppose the number of combinations of n things taken r together to be equal to the number taken $r+1$ together, and that each of these equal numbers is to the number of combinations of n things taken $r-1$ together as 5 is to 4, find the values of n and r. (*Todhunter.*)

The combinations which are equal in number must be complementary;

$\therefore n = r + r + 1$

$= 2r + 1.$

$\therefore \frac{\lfloor n}{\lfloor r \rfloor r+1} = \frac{5}{4}\frac{\lfloor n}{\lfloor r-1 \rfloor r+2}$

$\frac{\lfloor r-1 \rfloor r+2}{\lfloor r \rfloor r+1} = \frac{5}{4}$

$\frac{r+2}{r} = \frac{5}{4}$

$1 + \frac{2}{r} = \frac{5}{4}$

$\frac{2}{r} = \frac{1}{4}$

$\therefore r = 8.$

$\therefore n = 2r + 1$

$= 17.$

6. DUFFER.—If the number of combinations of n things taken $r - r^3$ together be equal to the number of combinations of n things taken $r + r^3$ together, find n. (*Todhunter.*)

If you refer to Articles 495 and 496, you will see that the combinations must be *complementary*:—

Thus $n = r - r^3 + r + r^3$

$= \underline{2r.}$

7. J. O. K.—Find the number of the combinations of the letters in the word *proportion* taken 6 at a time. (*Todhunter.*)

(*a*) There is 1 case in which no letter is repeated.

(*b*) There are 5 cases in which p occurs twice, as many in which r occurs twice, and as many in which o occurs twice; 15 in all.

$$\left(\frac{5 \cdot 4 \cdot 3 \cdot 2}{\lfloor 4} = 5\right)$$

(*c*) There are 10 cases in which o occurs three times.

$$\left(\frac{5 \cdot 4 \cdot 3}{\lfloor 3} = 10\right)$$

(*d*) There are 6 cases in which p and r each occur twice, as many in which p and o each occur twice, and as many in which r and o each occur twice; 18 in all.

$$\left(\frac{4 \cdot 3}{\lfloor 2} = 6\right)$$

(*e*) There are 4 cases in which o occurs three times and p twice, and as many in which o occurs three times and r twice; 8 in all.

(*f*) There is one case in which p occurs twice, r twice, and o twice.

∴ Number of combinations = 1 + 15 + 10 + 18 + 8 + 1
= 53. Ans.

Arithmetic.

1. R. S. L.—If in a field, where the grass grows uniformly, 62 oxen consume $17\frac{1}{2}$ acres in three-fourths of the time in which 15 oxen consume $5\frac{1}{4}$ acres, and 11 oxen takes 3 days more to consume $3\frac{3}{4}$ acres than 40 oxen do to consume $12\frac{1}{2}$ acres, find the time in which the 62 oxen consume the $17\frac{1}{2}$ acres, the original grass on each acre being supposed to be the same. (*Oxford and Cambridge School Examinations*, 1887.)

Let N = number of oxen fed by the grass growing on each acre for any length of time;
Then,

$$\left.\begin{array}{l}(62 - 17\frac{1}{2}\,N) \text{ oxen} : (15 - 5\frac{1}{4}\,N) \text{ oxen} \\ 5\frac{1}{4} \text{ acres} : 17\frac{1}{2} \text{ acres}\end{array}\right\} :: 4 : 3$$

$$(62 - 17\frac{1}{2}\,N) \times 5\frac{1}{4} \times 3 = (15 - 5\frac{1}{4}\,N) \times 17\frac{1}{2} \times 4,$$

$$(N \times 5\frac{1}{4} \times 17\frac{1}{2} \times 4) - (N \times 17\frac{1}{2} \times 5\frac{1}{4} \times 3) = (15 \times 17\frac{1}{2} \times 4) - (62 \times 5\frac{1}{4} \times 3),$$

$$N \times \frac{21}{4} \times \frac{35}{2} = 1050 - \frac{1323}{2} = \frac{147}{2},$$

$$\therefore N = \frac{\overset{1}{\cancel{147}}}{\underset{1}{\cancel{2}}} \times \frac{4}{\underset{1}{\cancel{21}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{35}}} = \frac{1}{5} \text{ ox.}$$

Let T = number of days in which 40 oxen can consume $12\frac{1}{2}$ acres;
Then,

$$\left.\begin{array}{l}\left(11 - \frac{3}{5} \cdot \frac{35}{4}\right)^{3} \text{ oxen} : \left(40 - \frac{2}{5} \cdot \frac{35}{2}\right) \text{ oxen} \\ 12\frac{1}{2} \text{ acres} : 3\frac{3}{4} \text{ acres}\end{array}\right\} :: T : T + 3$$

$$\left.\begin{array}{l}8 \text{ oxen} : 30 \text{ oxen} \\ 50 : 15\end{array}\right\} :: T : T + 3$$

$$(T + 3) \times 8 \times 30 = T \times 30 \times \frac{35}{3},$$

$$8T + 24 = 9T,$$

$$9T - 8T = 24,$$

$$\therefore T = 24 \text{ days.}$$

$$\therefore \left.\begin{array}{l}\left(62 - \frac{2}{5} \cdot \frac{7}{2}\right) \text{oxen} : (40 - \frac{2}{5} \cdot \frac{35}{2}) \text{ oxen} \\ 12\frac{1}{2} \text{ acres} : 17\frac{1}{2} \text{ acres}\end{array}\right\} \overset{\text{days}}{::} 24 : \text{Time reqd.}$$

$$\left.\begin{array}{l}48 \text{ oxen} : 30 \text{ oxen} \\ 25 : 35\end{array}\right\} \overset{\text{days}}{::} 24 : \text{Time.}$$

$$\therefore \text{Time} = \frac{\overset{3}{\cancel{30}} \times \overset{7}{\cancel{35}} \times 24}{\underset{1}{\cancel{48}} \times \underset{1}{\cancel{35}}} \text{ days}$$

$$= \underline{21 \text{ days.}} \text{ Ans.}$$

2. W. R. SEYMOUR.—Assume that 6 men can do as much work in an hour as 7 women, and 8 women as much as 11 boys, and that 5 men can do a certain piece of work in 10 hours. How long will it take one man, 2 women, and 3 boys together to do the same piece of work? Express the result decimally. (*London Matriculation*, June, 1880.)

7 women can do as much as 6 men,

$$\therefore 2 \text{ ,, ,, ,, } \frac{6 \times 2}{7} \text{ men}$$

$$= \frac{12}{7} \text{ ,, ;}$$

11 boys ,, ,, 8 women

$$= \frac{6 \times 8}{7} \text{ men,}$$

$$\therefore 3 \text{ ,, ,, ,, } \frac{6 \times 8 \times 3}{7 \times 11} \text{ men}$$

$$= \frac{144}{77} \text{ men;}$$

∴ 1 man, 2 women, and 3 boys can do as much as

$$(1 + \frac{12}{7} + \frac{144}{77}) \text{ men}$$

$$= \frac{77 + 132 + 144}{77} \text{ men}$$

$$= \frac{353}{77} \text{ men;}$$

$$\therefore \overset{\text{men}}{\frac{353}{77}} : \overset{\text{men}}{5} :: \overset{\text{hours}}{10} : \text{Time}$$

$$= \frac{50 \times 77}{353} \text{ hours}$$

$$= 353)3850(10{\cdot}906515$$

$$\begin{array}{r} 353 \\ \hline 3200 \\ 3177 \\ \hline 2300 \\ 2118 \\ \hline 1820 \\ 1765 \\ \hline 550 \\ 353 \\ \hline 1970 \\ 1765 \\ \hline 205 \end{array}$$

$$= \underline{10.906515\ldots \text{ hours.}} \text{ Ans.}$$

Note.—(*a*) Please refer to the Rules. (*b*) Those papers cannot be sent by Book Post.

3. A LADY STUDENT. — Determine the longest paying voyage, when freight is a penny a mile for 10 tons, of a steamer carrying 2,000 tons of coal and cargo, supposing the steamer to go 10 knots (miles) an hour with a consumption of 60 tons of coal a day, the coal costing 12s. a ton, and the steamer £20 a day for wages, repair, and interest on capital.—(*Matriculation*, June, 1887.)

Daily expenses = (12s. × 60) + £20
= £36 + £20
= £56;

Daily charge for 10 tons = (10 × 24)d.
= £1;

∴ Cargo must exceed 560 tons to make a profit;

∴ Voyage must be less than

$$\frac{2{,}000 - 560}{60} \text{ days.}$$

$$= \frac{1{,}440}{60} \text{ days.}$$

$$= \underline{24 \text{ days.}}$$

Difference of a day's voyage gives a difference of $\frac{80}{120}$, or £6, in the daily profit;

Profit on a voyage of 23 days = £6 × 23
= £138;

,, ,, ,, 22 ,, = £12 × 22
= £264

,, ,, ,, 21 ,, = £18 × 21
= £378;

As the difference in the daily increase of profits = £12, the profits increase until the 12th day;

∴ Longest paying voyage is 12 days, and profit

= (£6 × 12) × 12
= £72 × 12
= £864.

4. E. V.—If a snail, on the average, creep 2 ft. 7 in. up a pole during 12 hours in the night, and slip down 16 in. during the 12 hours in the day, how many hours will he be in getting to the top of a pole 35 ft. high? (*Barnard Smith.*)

The snail advances at the rate of (2 ft. 7 in. − 16 in.), or 15 in., in every 24 hours, except for the part left after the last day; 35 ft. ÷ 15 in. = 35 ft. ÷ 1¼ ft. = 28.

As the snail creeps up 31 in. during the night, 30 in. remain after 26 days.

Time to creep up the last 30 in. $= \frac{30}{31}$ of 12 hrs.
$= \frac{360}{31}$ hrs.
$= 11\frac{19}{31}$ hrs.;
∴ Total time = 26 days $11\frac{19}{31}$ hrs.
= 635 $\frac{19}{31}$ hrs. Ans.

Mensuration.

1. SIGMA.—One side of a triangle is half the sum of the two others; the sum of the three sides is 84 feet, and the area of the triangle is 14 square yards; find the sides.—(*Moffatt's* 'Pupil Teacher's Course, Fourth Year.')

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c represent the sides, and $s = \frac{1}{2}$ sum of the three sides;

Now, $s = \frac{a+b+c}{2}$

$= \frac{a}{2} + \frac{b+c}{2}$

$= \frac{a}{2} + a$

$= \frac{3a}{2}$;

$\frac{3a}{2} = 42$,

$\therefore a = 42 \times \frac{2}{3}$

$= 28$.

$\frac{b+c}{2} = a$

$= 28$,

$b + c = 56$,

$\therefore b = 56 - c$.

Area $= \sqrt{s(s-a)(s-b)(s-c)}$,

$14 \times 9 = \sqrt{42(42-28)[42-(56-c)](42-c)}$

$= \sqrt{42 \times 14 \times (c-14)(42-c)}$

$= 14\sqrt{3(c-14)(42-c)}$,

$9 = \sqrt{3(c-14)(42-c)}$,

Squaring; $81 = 3(56c - c^2 - 588)$

$27 = 56c - c^2 - 588$

$c^2 - 56c = -588 - 27$

$= -615$

$c^2 - 56c + (28)^2 = 784 - 615$

$= 169$

$c - 28 = \pm 13$

$\therefore c = \pm 13 + 28$

$= 41$, or 15.

∴ Sides = 15, 28, and 41 feet.

General.

1. MAGISTER.—The amount of the pension depends on the number of years calculated for the expectation of life, and should be stated in the question.

The amount for expected lives of 65 years = £260, and is obtained from the following equation:—

$$\frac{1{\cdot}05^{(65-45)} - 1}{1{\cdot}05^{(65-45)} \times {\cdot}05} \times 300 = \frac{1{\cdot}05^{(65-40)} - 1}{1{\cdot}05^{(65-40)} \times {\cdot}05} \times A,$$

or, $1{\cdot}05^{25}(1{\cdot}05^{20} - 1) \times 300 = 1{\cdot}05^{20}(1{\cdot}05^{25} - 1) \times A$.

2. CYCLOPS.—The solution of your query appeared in our *last* issue.

3. EDINBURGENSIS.—See Rule 2 of Query Column.

4. FUGUE.—You have evidently copied the question incorrectly, as a sketch of a field cannot be given from the particulars. The total area is 5·03822 acres.

Will you kindly repeat your query?

5. PRACTICAL.—The instruments supplied at the University at the Intermediate Examination in Science vary with the exercises to be performed. You need not take anything with you. We believe you can take nearly anything of your own that you choose. If you have any favourite scissors, scalpels, or glasses, you can take them, good microscopes are always provided. If you wish it we can tell you the actual practical work given this July. The apparatus supplied can form no guide to what you should buy. You are supposed to study the whole of the syllabus while you will only be examined upon a part of it.

6. BOROUGH ROAD COLL.—The proof is very simple, and you must notice that the two squares are equivalent to the square C K F H.

Triangles H B C, F E K, K D C, F G H can be all proved to be equal by (I. 4), and figure C K F H can be proved to be a square by I. 29 and 32.

If you cannot solve it after the above explanation, please write again.

7. ROMAN.—The large-hand is only fair. The strokes should be bolder and firmer, the curves more even and regular, 65. The small-hand is better. The letter *r* is of the large-hand form, only smaller; both the large and small are too upright. The capitals are bad, 60. Buy about three copy books and write for a few minutes a day for three months. For a classification of English novels refer to *Dr. Angus'* History of English Literature, near the end of the volume. If you want one volume you could hardly do better than read Westward Ho, by *Charles Kingsley*.

8. ET TU BRUTE.—Large-hand. The forms are nearly all incorrect. Only one letter *s* is correctly shaped. There seems no reason why you should not write well if you had some good copies and imitated them, 50.

Small-hand. The faults are mainly the same, but not so marked. The remedy is exactly the same, 60.

9. A.C.B.—You will have seen a paper on the subject of the Matriculation Examination in this magazine. You would do well to read the articles that have appeared. You do not state your age. We fear you would have to work some years before you could present yourself with reasonable hope of success. The languages themselves would take a considerable time, and it would be advisable to get some assistance. Would it not be better to study the subjects for the Certificate Examination and get into a higher class. If you are successful you would then be encouraged to matriculate.

10. C. W. GALE.—We have not the book you name by us. The answers you send us are quite correct.

(*a*) The balloon has been ascending vertically at a *uniform* rate for 4·5 seconds and then stops.

Let x = this rate. Then the height to which it has ascended is $4{\cdot}5x$. A stone is now allowed to fall, and it reaches the ground in 7 seconds. Neglecting the resistance of the air, the space through which a stone falls in 7 seconds is given by the formula:

$S = \frac{1}{2}gt^2$; therefore the height $2\frac{1}{2} \times \frac{32}{1} \times 4^2 = 784$ ft.

$\therefore 4{\cdot}5x = 784$ = the height

$\therefore x = 174\frac{2}{3}$ = the velocity.

(*b*) If the balloon is still ascending the stone would be moving vertically upward with the same uniform velocity as the balloon call this y.

Then the height of the balloon would be $4{\cdot}5y$. The space through which the stone moving vertically upwards with a velocity of x feet per second would fall in 7 seconds $= \frac{1}{2}gt^2 - 7y$

Therefore $4{\cdot}5y = \frac{1}{2} \times \frac{32}{1} \times 4^2 - 7y$

$11{\cdot}5y = 784$

$y = 68{\cdot}17$ = velocity

$4{\cdot}5y = 306{\cdot}76$ = height.

11. W.O.R.—For the first two try *Arber's* reprints. You will have no difficulty in procuring editions of *Gray* and *Burns*, order any. There is an edition of Burns' Songs by *Shipsey*, 1s. The prose prefaces are not published separately so far as we know. You will in that case be obliged to get the volume containing the preface of a really good edition. We believe there is a very fine edition by Bell and Daldy.

12. EAST ANGLIA.—There used to be a book of about fifty pages (1/6) of the kind you mention. It is many years since we saw a new copy of it. It may be out of print. It was published by Longmans & Co. Tell your bookseller to enquire for it.

13. J. G. GRIFFITHS.—Much depends upon the use you intend to make of them. There is a good book by *Ricks*, one by *Hassell*, and many more. There can be no clearer or better notes of lessons than those which have appeared and are still appearing in this magazine. Study them. You have not stated your age or the test you wish us to apply to your writing. It is of a very elementary kind. Judged by our usual standards we should call the large hand very poor and the small hand very bad. It is legible, that is its only merit; not one to be despised. Large hand 35, Small hand 30.

14. JEAN.—Please consult a work on the violin.

—o—

Helps to the Study of Shakespeare's 'Hamlet.'

ACT I. Scene v. Lines 6, 7. *Bound to hear*,—ready to hear. In the north of England the word is commonly used for ready or prepared. In Old English and in provincial dialects at present the final *d* is lacking. In the reply of the Ghost 'bound' is understood after 'thou,' but as the past participle of *bind*.

Lines 10—13. *Doom'd for a certain term . . . . purged away*.—This refers to the doctrine of purgatory. *Doom'd*,—sentenced. For the phrase *to fast in fires*, 'to lasting fires' has been suggested. But in Chaucer's *Parson's Tale* occurs the sentence 'the misere of helle shal be in defaute of mete and drink.'

Lines 13—22. *That I am forbid . . . . to ears of flesh and blood*.—'Forbid' used for 'forbidden' in accordance with a common tendency to drop the *en* of old participles. 'Harrow up,'—distress: the figure is derived from the agricultural process of harrowing or breaking up the ground, as in Scene 1, 'It harrows me with fear and wonder.'

Freeze thy young blood—chill thy blood and make it curdle. *an end*, on end; 'an' is the same word here as the prefix *a* in the words *abed*, *aboard*, where it means on or in.

fretful porpentine,—irritating porcupine; 'fretful' is used with active meaning; 'porpentine' is the form always used by Shakespeare, though editors sometimes change it to porcupine. The more usual form of the name is derived from *porcus*, a pig, and *spina*, a thorn.

eternal blazon,—dreadful revelation; 'eternal' may here be used for 'infernal,' as in Julius Cæsar, 'the eternal devil,' or it may have reference to the eternity of the future state, or the mystery of the unseen world. 'Blazon' is a term in heraldry denoting the description of armorial bearings. It is derived from *blaze*; compare the phrase, 'to blaze abroad,' to report.

Lines 29—31. *Haste me to know't . . . . to my revenge*.—Let me know the matter speedily that I may hasten to revenge the wrong with the swiftness of imagination or a lover's fancy.

Line 32. *duller shouldst thou be*,—thou wouldst be slower, less active.

Line 33. *that roots itself on Lethe wharf*,—that grows on the banks of Lethe, the river of oblivion in Hades. In the first folio 'rots' occurs instead of 'roots,' and it seems equally appropriate.

Line 35. *sleeping in my orchard*.—This participial phrase relates to 'me' not 'serpent' as strict grammatical construction would suggest.

Lines 36—38. *so the whole ear, &c.*—So, by a false story as to the manner of my death, the ear of all Denmark is grossly deceived. 'forged process' probably means an invented official narrative of the circumstances—analogous to the French *procès verbal*.

Line 40. *O my prophetic soul!*—This is an allusion to Hamlet's earlier suspicions:—Scene 2. 'I doubt some foul play!'

Line 42. *Adulterate*,—adulterous; compare 'emulate' for emulous in Scene 1.

Line 52. *to those of mine*.—Compared with mine.

Line 53. *Virtue* is a noun used absolutely.

Line 55. *garbage*,—refuse. This word is cognate with *garble* from Spanish *garbillar* to sift, Latin *cribellum*, *cribrum*, a sieve. The noun means what is left after sifting.

Line 59. *Sleeping within my orchard*.—This participial phrase is used absolutely, that is, the construction does not show its actual relation to the sentence; it is equivalent to 'while I was sleeping.' 'Orchard' is corrupted from *wort-yard*, root-garden; *wort* being cognate with *root* and *ort*.

Line 60. *of the afternoon*.—The use of the preposition 'of' to denote time was formerly very common, and still remains in many phrases;—'of a week-day,' 'of yore,' 'of late.'

Line 61. *secure hour*—unguarded hour. 'Secure' is here used in its literal sense as expressing freedom from care, from Latin *se* for *sine*, without; *cura*, care. The accent is on the first syllable.

Line 62. *hebenon*.—This is often supposed to mean 'ebony,' but the reasons appear stronger for regarding it as meant for 'henbane,' which is a poisonous plant.

Line 64. *leperous distilment*,—extract obtained by distillation, and producing effects like the disease of leprosy, as described in the succeeding lines.

Line 68. *sudden vigour*.—Staunton suggests that 'rigour' is a more suitable reading, as expressing the more usual result of narcotic poisoning; still 'vigour,' may be correct as indicating violent spasmodic action.

posset and curd,—congulate, as milk mixed with wine or an acid; 'posset' is not elsewhere used as a verb.

Line 69. *eager droppings*,—'eager' is used in the sense of sour, like the French *aigre*, from which comes our *vinegar*.

Line 71. *instant tetter bark'd about*,—an instantaneous scab appeared on the body like bark on a tree; 'tetter' is applied to ringworm.

Line 72. *lazar-like*,—like a leper. 'Lazar' is derived from Lazarus in the parable, Luke xvi.; *Lazaretto* is a house for lepers.

Line 75. *dispatch'd*,—deprived, bereft. It is only to the word 'life' that 'dispatched' exactly applies, and then it would more properly be connected with 'from' instead of 'of.' It is used with the other nouns by the construction called *zeugma*. Notice how the idea of suddenness is conveyed in 'dispatched.'

Line 77. *Unhousel'd, disappointed, unaneled*—without receiving the Eucharist, without preparation, without extreme unction. *Housel*, or *husel*, was the Anglo-

Saxon for a sacrifice, and then for the Eucharist; the verb *huslian*, was derived from it. With this use of 'disappointed' compare the expression well-appointed, well-fitted, well-equipped. 'Unaneled' is from an old verb *anele*, to anoint.

Line 78. *No reckoning made*,—without having settled my accounts. The phrase is absolute.

Line 80. *O, horrible!*—Some editors suggest, very reasonably, that this line is meant for an exclamation of Hamlet.

Line 83. *luxury*,—sensuality. This word, which is always used by Shakespeare in its bad sense of wicked excess, has lost much of its bad meaning, and is applied to comforts of a costly kind.

Line 88. *Fare thee well.*—This is one of the many instances where Shakespeare uses 'thee' for 'thou.' Dr. Abbott explains it as due partly to euphonic reasons, and also to usage, adopting 'thee' where little or no emphasis is required. We have gone a step further in omitting the pronoun.

Lines 89, 90. *The glow-worm . . . . uneffectual fire.*—These exquisite lines contain two or three peculiarities; 'matin' is nowhere else thus used for morning, nor 'pale' as a transitive verb; 'uneffectual,' without effect, the light being unaccompanied by heat. This is an instance of the use of the English prefix *un* instead of the Latin *in*, not.

Line 94. *instant old*,—instantaneously old. Here 'instant' is an adverb. Compare line 71, where it is an adjective.

Line 97. *this distracted globe*,—his head, as containing his brain, excited with distracting thoughts. It has been suggested that 'globe' means the world, and that such meaning would fit well Hamlet's philosophic turn of mind; but the former explanation appears the more likely. In acting this it is usual for Hamlet to put his hand to his head, some actors tightly clasping the head with both hands.

Line 98. *table of my memory*,—Here memory is compared to the tablets upon which it was customary to write down memoranda, as now in a pocket book. Compare 'my tables,' line 107.

Line 99. *trivial fond records*,—slight and foolish notes. 'Trivial' is from the Latin *trivie*, three ways, a place where three ways meet, a common resort of people with nothing to do but gossip, or talk on trifling passing matters. 'Fond' is still commonly used for silly or foolish in Staffordshire and the neighbouring counties; 'records' accented on second syllable.

Line 100. *Saws*—sayings: saw and say are two modifications of the Old English verb *seggan*. Compare the Icelandic *saga*, a tale.

pressures,—impressions, the results of pressure, as of a seal.

Line 105. *pernicious*—destructive; literal meaning, killing utterly, from Lat. *per*, thoroughly; *neco*, I kill.

Line 106. *villain*,— a wicked person,—original meaning a person attached to a *villa*, or farm, a villager, afterwards, a serf, a person of degraded rank, then of degraded character. This is a striking example of the deterioration to which words are liable.

Line 107. *My tables,—meet it is I set it down.*—This is a matter proper to be noted on my tablets. The remark implies that making notes was a common practice of Hamlet's, and the note he makes betokens his tendency to cynicism.

Line 110. *So, uncle, there you are. Now to my word.*—This is said after the note has been made: 'My word' means my watch-word or motto; it consists of the last words of the Ghost before his disappearance.

Line 114. *Heaven secure him.*—Heaven guard him, make him secure.

Line 116. *Hillo, ho, ho, boy*,—A cry used in hawking, to recall the falcon.

Line 121. *once think it*,—ever think it.

Line 124. *arrant knave*,—thorough rascal, 'arrant' is said to be cognate with *arch*, cunning, derived from German *arg*, bad, crafty; 'knave,' from A. S. *cnafa*, a boy, a youth, German *knabe*, is another instance of the deterioration of meaning which words undergo.

Line 125. *come from the grave.*—Notice the omission of to before the infinitive after 'needs.'

Line 127. *circumstance*,—talking round about the matter, beating about the bush.

Line 132. *I'll go pray.*—Instance of *to* omitted. Compare 'go seek,' 'Come see,' and many more.

Line 137. *Touching this vision*,—With regard to this apparition, 'Touching' is a participle in process of becoming a preposition, like during, concerning.

Line 139. *For your desire.*—As for your wish. 'Desire' is from French *désirer*, It. *desirare*, Lat. *desiderare*, to desire, connected with *sidus*, *sideris*, a star, and the root of consider.

Line 142. *soldiers.*—This is a trisyllable. It is derived from Lat. *solidus*, silver money, pay; *solidatus*, one paid. This is an example of a word improved in meaning; the root-idea seems to suggest a mercenary, a hireling.

Line 149. *Indeed, upon my sword, indeed.*—Staunton suggests that 'indeed' might be made more emphatic by printing it as two words. It means not in word only, but in act, in truth. An oath on a sword was considered specially binding from the hilt being shaped like a cross. In a warlike nation like the Scandinavian, other associations would add weight to swearing by the sword.

Line 150. *truepenny*—honest fellow. The term seems familiar, but is not necessarily wanting in respect.

Line 151. *in the cellarage*—under ground.

Line 156. *Hic et ubique.*—Lat. for here and everywhere, in allusion to the change of place from which the Ghost's voice comes.

Line 162. *Old mole*—again referring to the change of place. 'Mole' is contracted from 'mould warp' the old name of the mole, meaning a thrower or caster up of mould or earth.

Line 163. *pioner*,—for pioneer, but accented on first syllable; from French *pionnier*, Old Fr. *peonier*, *peon*, Ital. *pedone*, a foot soldier, from Lat. *pes*, *pedis*, a foot; later meaning, one who clears the road before an army; Shakespeare uses 'enginer' for engineer. 'muleter,' and 'mutiner.'

Line 165. *As a stranger give it welcome.*—This may mean give it a hearty welcome, as you should a stranger, or treat it as a strange person, do not seem to know it.

Line 167. *in your philosophy.*—Do not emphasise 'your;' the pronoun is used colloquially and generally, not personally.

Line 172. *to put an antic disposition on*,—to assume a strange or odd manner. 'Antic' is a form of *antique*, with the meaning of old-fashioned, unusual, fantastic.

Line 174. *With arms encumber'd thus, or this head-shake,*—With arms folded like this, or with a significant shake of the head; 'encumber'd' is not used in this sense in any other place.

Line 176. *an if.*—An instance of reduplication, the two words being identical in meaning.

Line 177. *If we list,*—if we chose. Compare 'The wind bloweth where it listeth.' 'List' is from Anglo-Saxon *lystan*, to desire, German *lust*, pleasure. It is cognate with listless, lustful.

Line 178. *Ambiguous giving out,*—doubtful statement, insinuation. 'Ambiguous' is from Lat. *ambiguus*, driving about, from *ambigo*, to go about, from *ambo*, about, *ago*, I go, or drive. It means having more than one meaning.

to note,—'to' is needless, because note depends on 'shall.'

Line 180. *So grace and mercy at your most need help you,*—So may grace and mercy help you in the time of your greatest need.

Line 186. *his love and friending,*—'Friending' is a peculiar word used for 'befriending' in the sense of friendly disposition carried into action.

Line 187. *God willing, shall not lack.*—God willing, is grammatically an absolute phrase, expressing a condition: 'shall not lack' means shall not be wanting.

Act II., Scene 1.

Notice here the use of '*shall*' for 'will,' '*marvellous*' for 'marvellously,' '*inquire*' for 'inquiry;' '*Inquire me*,' that is 'for me,' dative; '*Danskers*,' Danes; '*encompassment and drift*,' going round about, driving to an end; *come you*, you will surely come; *more nearer*, double comparative; *Than your particular demands will touch it*, than you will attain by direct or special questions: 'It' used as an indefinite object for 'the matter.' *Take you*, take upon yourself, assume; *addicted*, inclined to, given up to; *put on him*, lay to his charge; *forgeries*, invented faults; *rank*, gross; *usual slips*, common venial offences; *season it*, temper or moderate it; *scandal*, disgraceful charge, Fr. *scandale*, Lat. *scandalum: breathe his faults so quaintly*, speak of his failings in low tones and in a delicate manner; 'quaintly' is often used for neatly, curiously; *unreclaimed blood*, spirit unsubdued; *of general assault*, such as commonly assail the young: *my drift*, what I am driving at; *fetch of warrant*, justifiable trick, or catch: *You laying*, &c., an absolute phrase; *converse*, conversation; *him you would sound*, he whom you would question,—he introducing absolute phrase; *prenominate crimes*,—before-named offences; *closes with you*, follows up the conversation; *addition*, title or form of address,—compare 'soil our addition,' Act I., Scene 4; *leave*, leave off; *a'gaming*, he. Mark the colloquial character of this supposed speech. *o'ertook in's rouse*, overcome in his drinking bout, a euphemism for intoxicated; *tennis*, derived from Fr. *tenez*, hold, Latin *teneo*, I hold; *we of wisdom and of reach*, we who are wise and far-sighted; *windlasses*, roundabout ways; *assays of bias*—trials of indirect aim or tendency, such as a player practises in the game of bowls, not sending the bowl in a straight line but calculating on the deviation caused by its bias; *indirections*, winding ways. *You have me*, you understand me. *Observe his inclination in yourself*, put yourself in his place, or judge him by your own feelings.

Doublet all unbraced, outer garment unfastened. *down-gyved*, hanging about his ancles like fetters, from Welsh *gefyn*, fetters. *purport*, meaning; the accent is on the second syllable. In this speech notice the absolute phrases used to describe the appearance of Hamlet.

Perusal, careful examination; the prefix *per* means thorough. *As* for 'as if,' and *as* for 'that.' *bulk*, according to some commentators this means body, according to others the bust or upper part of the trunk. *bended*, for bent, turned. *go seek*, compare 'I'll go pray,' Act I. Scene 5. *ecstasy*, madness, from Greek εκ out, στασις, a standing, standing out of oneself; compare the expressions, out of one's mind, besides oneself; *fordoes*, destroys, undoes; the prefix *for* has two meanings, negative in forget, forbear, forbid, forgo,—intensive in forgive, forlorn. *repel*, send back: *access*, approach, accent on first syllable. *quoted*, marked, scanned. *beshrew my jealousy*, a mild imprecation against his suspicion.

It is as proper . . . . lack discretion.—It seems as fitting for old men to make mistakes through excessive suspicion, as it is common for young men to go wrong for want of ripe judgment.

Might move more grief . . . . to utter love, might cause more distress through unnatural concealment, than the offence which might arise from the acknowledgment of affection.

(To be continued.)

—o—

Recent Inspection Questions.

Arithmetic.

STANDARD I.

1. 605 + 76 + 804 + 83 + 240. 1808. Ans.
2. 120 − 63. 57. Ans.
3. 736 − 195. 541. Ans.

(Dictation.)

House, little, duck, wave, school, head, window, friend, bread, life.

STANDARD II.

A.

1. 53,200 ÷ 7. 7,600. Ans.
2. 5,900 − 3,012. 2,888. Ans.
3. 58,976 × 409. 24,121,184. Ans.

B

1. 83,511 ÷ 9. 9,279. Ans.
2. 60,040 − 953. 59,087. Ans.
3. 8,764 × 894. 7,835,016. Ans.

STANDARD III.

A.

1. Multiply six huudred thousand eight hundred and ninety-five by eight hundred and nine and divide by twelve (short division).

486,124,055 ÷ 12 = 40,510,337$\frac{11}{12}$. Ans.

2. How much is £20,046 17s. 6d. greater than £1,090 18s. 5¾d. £18,955 19s. 0¼d. Ans.

3. Add £6 11s. 7½d., £536 19s. 2¼d., £7,123 10s. 9¼d., £846 10s. 7¾d., £8 19s. 3¾d.
£8,522 11s. 6¼d. Ans.

4. John buys a score and a half of oranges at a shilling a score, and sells them at 1d. each; how much does he gain? One shilling. Ans.

B.

1. £77 19s. 7½d. + £10,599 16s. 11¼d. + £76 9s. 0d. + £9,708 13s. 11¾d. + 19s. 2d.
£20,463 18s. 8½d. Ans.

2. Subtract £177 13s. 0½d. from £3,067 3s. 1¼d.
£2,889 10s. 0¾d. Ans.

3. Divide seven millions five hundred and nineteen by 438. $15,982\frac{103}{438}$. Ans.

4. A piano and music stool together cost £76 19s. 5½d. The stool cost 38s. 7¾d. How much less did the stool cost than the piano?
£73 2s. 2¼d. Ans.

STANDARD IV.

A.

1. How many hours in 10 years in which there are three leap years. 87,672 hours. Ans.

2. A farmer had five carts of potatoes, each containing 35 cwt. What was the crop worth if he had sold them all at 6d. a stone? £35. Ans.

3. £3 12s. 2½d. × 27. £97 9s. 7½d. Ans.

4. £4,047 8s. 11¾d. ÷ 26.
£155 13s. $5\frac{9}{13}$d. $\frac{7}{13}$. Ans.

B.

1. £86 15s. 3¾d. × 97.
£8,416 5s. 3¾d. Ans.

2. How many lbs. are 256,800 ozs. greater than 7 tons 3 cwts.?
16,050 − 16,016 = 34 lbs. Ans.

3. £925 3s. 2¾d. ÷ 95.
£9 14s. 9¼d. Ans.

4. If a boy lived 3 fur. 24 poles from school, how many yards did he travel to make 380 attendances if he went home to dinner each day.
3 fur. 24 yds. = 684 yds., distance to school. 684 × 380 × 2 = 519,840 yds. Ans.

STANDARD V.

A.

1. Make a bill for 7 doz. at 2½d. each, 31½ yds. at 6d. per yd., 14½ yds. at 2s. 7d. per yd., and five doz. at 3 for 2d.
(17s. 6d. + 15s. 9d. + £1 17s. 5½d. + 3s. 4d.)
= £3 14s. 0½d. Ans.

2. Take $\frac{3}{5}$ from $\frac{8}{9}$ and add $\frac{1}{6}$.
$(\frac{8}{9} - \frac{3}{5} + \frac{1}{6}) = \frac{7}{18}$. Ans.

3. How many horses at £25, are equal to 15 oxen at £15?
£25 : £15 :: 15 : 9 horses. Ans.

4. A man sold half an acre at 1d. per sq. yd. He bought chickens @ 3s. 4d. a couple with the money; how many did he buy?
2,240d. ÷ 20 = 112 chickens. Ans.

B.

1. Find by practice the cost of 23 cwt. 3 qrs. 6 lbs. at 4s. 1½d. per lb. £549 17s. 3d. Ans.

2. I rent a field 10 weeks for 35s. Find how many days' rent 11 guineas will pay.
462 days. Ans.

3. Four pairs of gloves are worth 1½ yds. of silk at 4s. 6d. a yard. How much will be left out of a guinea after paying for a dozen pairs of gloves.
9d. Ans.

4. From the sum of $\frac{1}{2}$ and $\frac{2}{3}$ take the difference between $\frac{5}{6}$ and $\frac{2}{3}$.
$(\frac{1}{2} + \frac{2}{3}) - (\frac{5}{6} - \frac{2}{3}) = 1$. Ans.

STANDARD VI.

A

1. Reduce $1\frac{1}{3}$ of $\frac{8}{9}$ of 13s. 6d. to the dec. of £1 7s. 0d.
.3. Ans.

2. Find the simple interest of £367 2s. 6d. for 2½ years at 6 per cent. per annum.
£55 1s. 4½d. Ans.

3. Add ·705, 7·05 and ·00705 together and divide the result by 7·05. 1·101. Ans.

4. 200 yards of cotton are wound on a reel. A girl uses 6 inches of cotton to 3 inches of sewing. How many yards will be left on the reel after 70 girls have each done 1¼ yards. 25 yards. Ans.

B

1. If 4·125 cwt. of coffee cost 21 guineas, what will 19·25 cost? £102 18s. Ans.

2. What is the amount of 10 guineas for 40 years at 4⅓ per cent. simple interest? £29 8s. Ans.

3. Multiply the sum of $4\frac{1}{8}$ and $3\frac{1}{3}$ by ½ of $\frac{5}{17}$.
$1\frac{25}{136}$. Ans.

4. Four persons start on a pleasure trip, each having £20, and agreeing to share their expenses equally. On returning one has spent all his money, and each of the others has £1 15s. 4d. left. What should each of the three give to the one who has spent all to make things right? 8s. 10d. Ans.

STANDARD VII.

1. A tradesman's annual average gain for the years 1870 to 1874, both inclusive, was £196 10s. 11d. In 1870 he gained £50; in 1874 £228 10s. 5d. What was the annual average for the other years?

£234 14s. 8$\frac{2}{5}$d. Ans.

2. If 288 threepenny loaves serve 36 men for 12 days how many fourpenny loaves will serve 42 men for 18 days?

Loaves.

Men 36 : 42 :: 288 : 378. Ans.
Pence 4 : 3
Days 12 : 18

3. If by selling oranges at 1d. per doz. I gain 10 per cent., what will be the price per doz. if I gain 25 per cent.

110 : 125 :: 1 : 1$\frac{3}{22}$d. Ans.

4. A sum of money is lent at 2$\frac{1}{2}$ per cent. simple interest, how long will it take to get ·8 of the principal. To get the principal would require—

100 ÷ 2$\frac{1}{2}$ = 40 years.

To get ·8 would take $\frac{4}{5}$ of 40 = 32 yrs. Ans.

How to teach the Babies.

BY JAMES BAILEY.

Head Master of Southlands Training College.

CHAPTER IX.

'NUMBER.'

MR. CURRIE, in his admirable work 'Early and Infant School Education,' makes the following remarks on the teaching of number:—'Number is a property of things which the child observes very early, so that he may be exercised upon it as soon as he enters the infant school.' 'As the child comes by his first notions of number through the medium of objects, so his whole training must be based on the observation of these. He does not use numbers for their own sake, but for the sake of the things to be numbered; he counts by sight, and is not able to abstract number from the things. He knows what five balls or five horses are, but he cannot reason about the number five. If it be understood that it is with number as a *property of bodies* that the infant has to deal, and not with the science of number, it will be very clear that he must not be occupied with rules or technical operations. This preliminary course of training is termed "on number," to distinguish it from the formal study of arithmetic.'

Recognising, accordingly, that we deal with number as a 'property of things,' the early exercises must be invariably associated with things, *i.e.*, to put it technically, we deal with it 'in the concrete,' and not at first 'in the abstract.' If this be borne in mind and sensibly applied, and if, further, we speak of it in simple and familiar language, we may as safely and as naturally place number among the earliest school exercises, and therefore in the prescribed course for 'The Babies,' as *form* or *colour*, or any other property of things. This method of teaching number has long been recognised and provided for as a feature of well-arranged early teaching. The 'ball frame,' sometimes rather pedantically termed the 'abacus,' has formed a part of the apparatus of every well-appointed infant school from the time at least of the earliest efforts of Wilderspin and Stow in this direction.

Pestalozzi and Froebel give *number* a prominent place among the things to be learnt from the exercise of the children's senses upon the objects presented to them. A properly constructed ball frame is large enough to be easily seen by all the children in the class; it has ten wires, with ten balls strung on each wire, or, more conveniently, twelve wires with twelve balls on each, thus corresponding with the multiplication table to twelve times twelve. Properly used it may be both an interesting and an efficient aid to the teaching of this important subject. A sliding board or shutter covers one half of the length of the wires, and so can be used to shut off from the view of the class the whole or any number of the balls. This simple expedient enables the teacher to exhibit, on the free portion of the wires, just the number of balls which are needed for the required illustration, and thus prevents the distraction caused by having all the balls in sight at the same time. It must, of course, be borne in mind that there is no inherent virtue in the ball frame as such. One hears occasionally of 'lessons on the ball frame,' and sees it so handled and dealt with as though it were the subject of an object lesson, a knowledge of which would necessarily be followed by a knowledge of number. It is with this, as with the 'box of form and colour,' the gifts and materials of the Kinder Garten, and all other appliances and apparatus of the sort, its value depends entirely on the method in which it is used. It is a useful and very convenient expedient for illustrating, in a systematic form, the elementary knowledge of numbers and their combinations, and of the simpler processes of arithmetic. But it is nothing more than this.

Material illustrations of number will, of course, not be confined to the use of the ball frame. They will be found in the contents of the form and colour box, the Kinder Garten materials, objects of all kinds within the children's notice, and especially should the chalk and blackboard be freely employed for the same purpose.

As a general guide to the combination of good principle and practice in the early teaching of arithmetic, I know no better method than that suggested in the Section on Number, in Currie's 'Early and Infant School Education,' from which I have already quoted.

The first efforts must be directed to the teaching of the meanings and the names of the several numbers up to *ten*. One number only must be taken at a time, and each must be dealt with separately, beginning with *one*.

Thus: first show in succession a number of single things, naming them, thus; 'one finger,' 'one slate,' 'one boy,' 'one girl,' 'one book,' &c. Then, drawing each on the board as named, 'one straight stroke,' 'one square,' 'one circle,' 'one triangle,' placing before each the figure 1 (a short straight stroke). The little ones should follow this practically, repeating the action and the words of the teacher throughout in like manner. Each one, imitating the teacher, should lift a finger, and say 'one finger;' touching in succession the head, nose, mouth, desk, slate, book; pointing to door, window, blackboard, table, ceiling, &c. With each act, say 'one head,' 'one nose,' 'one door,' &c.

The teacher should next explain simply that the straight stroke, or figure 1, is named *one*, and that it means the *number* of the thing, as another word may mean the *shape* or the *colour* of the thing. So that *door*, *brick*, *slate*, *boy*, &c., are all alike in being *one* of the things they mean. The ball frame should be used from the beginning, but with the remembrance of what has been previously said, that there is no virtue in it beyond that of being a convenient means of indicating in an interesting way the number referred to.

When number One has been made as plain to the children's intelligence as the matter admits of, deal similarly with number Two. As before, hold up two fingers; let the children do the same, repeating after the teacher, 'two fingers.' So with 'two eyes,' 'two ears,' 'two lips,' 'two hands,' 'two books,' and so on. Let one child, or several children in succession, place out two balls on the frame, two pencils, or two small bricks on the desk, point out any two similar objects, &c. The figure 2 should be plainly marked on the blackboard, its name given, its meaning told, and it should be illustrated by 2 strokes, two dots, &c., each prefigured by the sign 2. Test and revise as before, by requiring children to point out, or place on desk, ball frame, or table, two things, and naming the number of them on each occasion.

A similar course should be followed with each number up to ten. There must be constant revision of previous lessons, frequent recapitulation by the children, patient reiteration by the teacher of what they have forgotten or imperfectly apprehended, and an unceasing cheerfulness and simplicity pervading every lesson.

During the process of learning the values of the numbers up to *ten*, some knowledge of *addition* will be incidentally acquired. But this stage being reached, simple lessons on addition and subtraction should be systematically given.

Addition should begin with *one* having added to it successively, *one*, then *two*, *three*, and so on, but leaving no step till pains have been taken by many illustrations to make it perfectly clear. Then, placing out one ball on the frame, put at a little distance from it, or on the wire below it, another ball; we now have one ball and one ball. We want a name for both. They are *two* balls. Do the same with one slate or book, and another slate or book, on the desk; one stroke on the board and another. Let one boy stand before the class; put another near him. In each case let the class repeat after the teacher, 'One ball and one ball are two balls;' 'one book and one book are two books;' 'one boy and one boy are two boys.'

It must be remembered that next to the value of *knowing* is the value of the clear and accurate *expression* of the knowledge. Hence, after teaching any simple point, let the simple statement of the fact learnt be repeated by the class. To ensure this, and to fix it in the children's minds, simple questions should follow the teaching of every step. Thus, 'What are one and one called?' 'How many ones are there in two?' 'How many more boys are two than one?' 'What do we mean by two slates, two houses, two cherries?' &c.

When this is fairly known, proceed in a similar way to teach 'one and two,' keeping the one stroke, book, boy, separate. Add to each two strokes, two books, two boys, &c., in each case saying as before, 'One stroke and two strokes are three strokes; 'One book and two books are three books,' 'One boy and two boys are three boys.' Let the class, with any objects at hand, or let one or more come out before the class and, themselves, make up the three by adding the two to the one.

The repetition of the simple formulas, the testing and revising by questions, and other forms of recapitulation, must follow in due order. The same patient detail must characterise the successive additions from 'one and one,' to 'one and nine.'

The analysis of the numbers successively taught will lead incidentally and directly to the teaching of,

Subtraction.—Placing out upon the ball frame in succession, but dealing with each separately, two, three, four, &c. Proceed thus:—'Take one from two, what is left?' 'Take two from three, what is left?' 'Three from four?' &c. Show and repeat, 'One from two leaves one.' 'Two from three leaves one.' Three from four leaves one,' &c. After removing a *variable* number in succession, take away a series of *constant* numbers, showing it in each case by *visible* illustration—'One from two leaves one,' 'One from three leaves two,' 'One from four leaves three,' &c.

Next, combine the exercises of Addition and Subtraction: thus; marking on the board, or placing balls on the frame, say 'One ball (or stroke), and one ball (or stroke), and one, and one, and one, and one,' 'How many balls?' 'If one is taken away, how many are left?' 'If two?' 'or three?' &c. 'If the three are put back, how many have we?' 'Three and three make six,' 'Three and two make five,' 'Three and one make four.' Show again the various ways in which six may be made up. 'One and five make six,' 'Two and four make six,' 'Three and three make six,' 'Four and two make six,' 'Five and one make six.' Deal similarly with other numbers until the analysis and the combinations of the several numbers shall be clearly understood.

Again, in the case of numbers made up of even quantities. Show the numbers of twos, or of other equal parts, of which the numbers are made up. In four, two twos; in six, three twos, or two threes; in nine, three threes; in ten, five twos, or two fives, &c.

Let it not be supposed that this method involves a needless elaboration of details. Only those who are practically acquainted with the difficulties attending the intelligent teaching of very young children can judge of the imperfectness of their appreciation of such matters as these, and of the mistakes they make in acquiring them. In respect of *number* this seems to be particularly the case. Certainly, patient and careful simplicity in the ground work is an essential condition of a correct and clear understanding of subsequent processes.

For the very young children comprised in our 'babies' classes,' the course suggested will probably be sufficient. The number *ten*, forms a convenient limit for the first stage. If however, it should be thought expedient to go beyond it in any case, the instruction should be continued on similar lines.

It has not been thonght necessary to indicate the precise limits of each lesson. The teacher only, can judge of what is fitting in each case. The lesson should be *short*; certainly never continued beyond the point at which the *interest* and therefore the *useful* attention of the children can be sustained by the teacher. Each lesson should comprise a simple, definite, piece of instruction. It should be preceded by a revision of previous lessons, and should be followed by an equally simple and careful revision of the ground it has covered.

Knitting Maps.

BY MRS. J. D. HARRIS.

BABY'S FIRST SHOE.

¼ oz. of Berlin wool.

Pins No. 15. Chamber's bell gauge.

Cast on one pin, 48 stitches.

Knit 2, purl 2 for one inch.

To begin the pattern.

*A. Slip the first stitch, knit 2 together for the rest of the row. Knit one stitch at the end of the row.

Knit the reverse row, picking up and knitting the loop between each stitch, and also putting the wool forward before the last stitch (otherwise you would lose one stitch).

Repeat these 2 rows till you have 9 rows of small holes.

B. Make a row of larger holes in which to run a ribbon. This is done by putting the wool forward and knitting 2 together. Knit 1 plain row.

C. Knit the pattern till you have 3 more rows of small holes.

To commence the Instep.

D. Knit 17 and put 17 from the other end on another pin.

E. On the instep pin you should have 14 stitches. Continue the pattern on this pin till you have 12 rows of holes from B.

To commence the Toe.

F. On the same pin knit plain backwards and forwards 10 rows. In the 10th row knitting the 2 first and the 2 last stitches together.

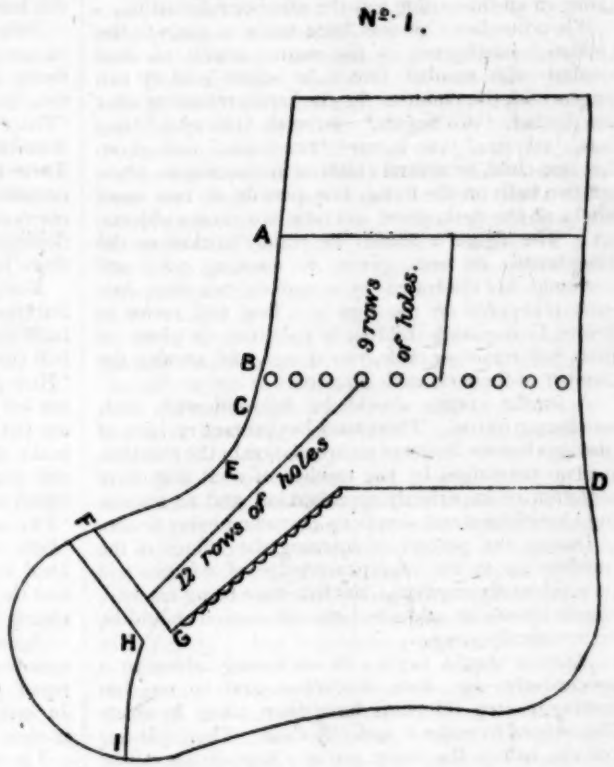
To form the Shoe.

G. On another pin pick up 13 stitches at the side of the instep, and knit the 17 stitches on to it. Knit back, and pick up 13 stitches the other side. You should have 30 stitches on each instep pin, and 12 on the toe pin.

Knit 8 plain rows.

To turn the Toe.

H. On the right side, when you come to the toe pin, knit the 2 first and the 2 last stitches together. Knit to the end of the heel, and back to the other side. Repeat once more. Then to round the heel knit the *2 first* stitches together of the 1st instep pin, and the *2 last* of the second instep pin. These reducings must always be on the right side. The reverse row is plain.



I. When the toe is reduced to one stitch knit the sole of the foot together and cast off.

The ankle should be neatly sewn on the wrong side, and a ribbon run in the large holes.

SOCK NO. 2.

FOR INFANT OF THREE MONTHS.

Cast on 56 stitches.

A. Knit 12 rows of holes.

B. Make a row of larger holes.

C. Knit 5 more rows of small holes.

D. 20 stitches on each side pin. 16 on instep pin.

E. Knit 16 rows of holes from B.

F. Knit 14 plain rows.

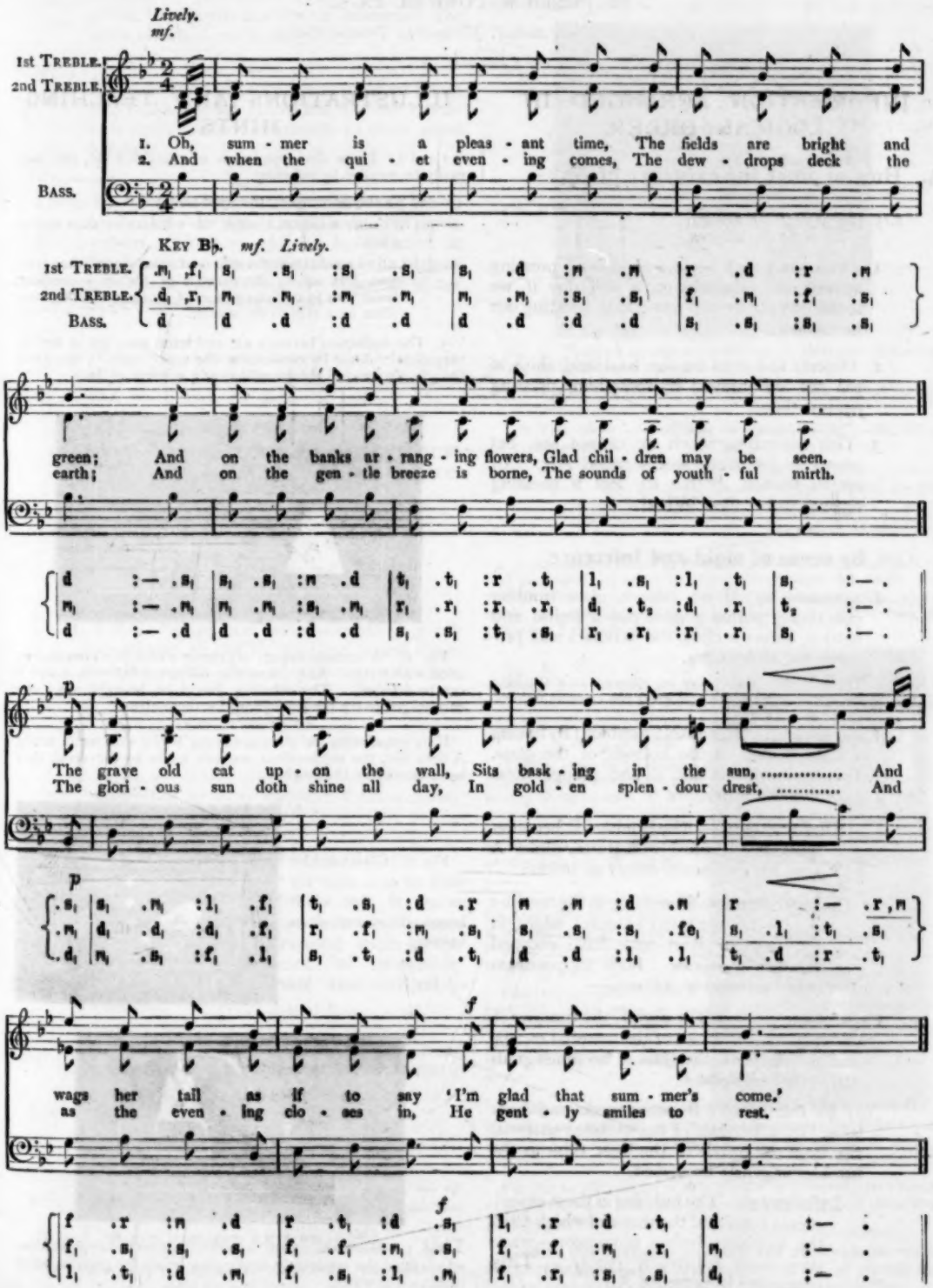
G. Pick up 16 stitches.

Knit 10 rows.

The reducings are exactly the same as the Shoe No. 1.

The ribbing at the top should be a few rows longer.

(To be continued.)

SUMMER.

Lively.
mf.
1st TREBLE.
2nd TREBLE.
BASS.
1. Oh, sum - mer is a pleas - ant time, The fields are bright and
2. And when the qui - et even - ing comes, The dew - drops deck the
KEY B♭. mf. Lively.
green; And on the banks ar - rang - ing flowers, Glad chil - dren may be seen.
earth; And on the gen - tle breeze is borne, The sounds of youth - ful mirth.
p
The grave old cat up - on the wall, Sits bask - ing in the sun, ... And
The glori - ous sun doth shine all day, In gold - en splen - dour drest, ... And
f
wags her tail as if to say 'I'm glad that sum - mer's come.'
as the even - ing clo - ses in, He gent - ly smiles to rest.

Notes of a Lesson on the Atmosphere.

AIR:—Its existence, extent, and most common properties.

BY JOSEPH H. COWHAM, F.G.S.,

Lecturer on School Method: Westminster Training College.

INFORMATION ARRANGED IN LOGICAL ORDER.

A. How to prove the existence of Air:—

(*a*) By sense of touch.

1. On a windy day we feel something pressing against us: Similarly on a still day if we move quickly we feel something resisting our movement.

2. Objects like trees on the land and ships at sea are also moved by something pressing against them.

3. This something which we cannot see, but which we frequently feel is called **Air**. Air set in motion so that we feel it pressing against us is called **Wind.**

(*b*) By sense of sight and Inference.

Experiment 1. If we take a glass tumbler containing neither a solid nor a liquid and hold it before a class, the scholars will pronounce it *to be empty*.

NOTE.—This common error may be allowed as a teaching device: proceed however at once to correct the error as follows.

Experiment 2. Now take a tumbler (B) having a hole drilled in the bottom of the glass. Invert tumblers A and B and plunge them gently under water.

Observation:—The tumbler B becomes immediately filled with water, whilst A remains apparently empty as before.

Supposition:—Something in the tumbler A prevents the water entering, whilst in the case of B it may have escaped through the hole. Both suppositions may be tested as follows:—

Experiment 3. (*a*) Tilt slightly the tumbler A under water. Bubbles of the material escape and as they leave the glass it becomes gradually filled with water.

(*b*) Fit the tumbler B with a cork and bent tube, Fig. 3, bubbles of the escaping material rise from the nozzle of the bent tube as the tumbler is gently immersed.

Inference:—The bubbles in these experiments consist of the material which filled the tumblers before immersion—when they were apparently empty. **This material is termed Air.**

ILLUSTRATIONS AND TEACHING HINTS.

1 and 2. These are experiences of every day life, and may easily be recalled by referring

(*a*) To the direction of the wind to-day.

(*b*) Whether it aids or hinders the scholars on their way to school.

(*c*) Call to recollection the effects of some storm when trees were torn up by the violence of the air movement. Refer also to the experiences of a trip in a sailing boat when on a visit to the seaside.

3. The distinction between **air** and **wind** may not at first be very clear. Assist by contrasting the quiet state of the air in the schoolroom and the draught near a window or door.

Fig. 1. A remains apparently empty whilst B is immediately filled with water. At a distance no difference between A and B can be detected. The scholars should be brought near, and encouraged to try the experiment for themselves.

If by questioning the class as to why B fills with water whilst A does not, the suppositions are not made by the class, they must be told by the teacher.

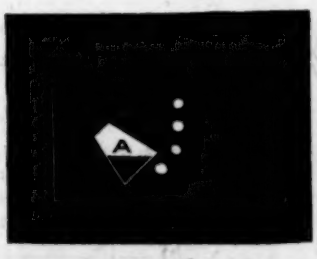


Fig. 2. Glass tumbler tilted so as to allow the escape of that which keeps the water from entering.

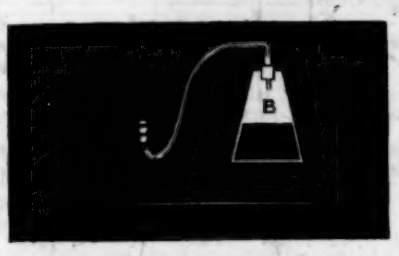


Fig. 3. Shows the escape in bubbles of that invisible material which filled the apparently empty glass when the glass is being immersed in water.

B. Extent of the Air.

1. As the above series of experiments can be successfully performed in any part of the globe, they prove that the air surrounds the earth on which we live.

2. If we ascend the highest mountain or go up in a balloon much above the highest mountain summit the air extends to these great heights.

3. Clouds have been shown in a previous lesson to be due to the atmosphere containing vapour. These clouds can be detected at heights two or three times that of the highest mountains thus proving the existence of air at very great heights indeed.

4. The air is estimated to extend at least to 50 miles above the earth, and in a very fine form may reach as much as to 200 miles.

The greatest amount of air however is within five miles of the earth's surface.

C. Most common properties of air.

1. Unlike water and other liquids a given quantity of air can be squeezed into a much smaller space than it ordinarily occupies. Hence:—

Air is compressible.

2. When air which has been compressed is relieved of its extra pressure it immediately occupies the volume which it filled before compression. Hence:—

Air is elastic.

3. It can be shown by experiment that, **when a gas like air is pressed with twice its ordinary amount of pressure it occupies one half the ordinary volume.***

For explanation and illustration of the above truth, See Fig. 5, and its accompanying statement.

4. If now we continue to take away pressure the amount (volume) of air continues to increase, and this is maintained so long as we continue to lessen the pressure. Hence:—

Air possesses expansibility.

* This statement presumes that temperature remains the same for both cases.

1. An easy step in reasoning which the children will readily take.

2. Relate the incident of Glashier's famous balloon ascent to the height of 36,000 feet in 1861. The air was found at this immense height to weigh scarcely one fourth as much as on the surface and explain this by the assistance of the diagram below.

Fig. 4. Diagram to assist in realizing the thickness or density of the air at different altitudes.

A Represents the first 5 miles of atmosphere when the air is thickest.

B The shell of thinner air extending 10 to 15 miles above A.

C The *very thin* air in the higher atmosphere.

N.B.—The terms *thick* and *thin* are changed for the proper terms below.

1. A simple experiment to show this is to take a common syringe and plug up the nozzle after filling the cylinder with water, the piston cannot be pushed in. Remove the water and plug tightly again, and now, although no air can escape, the piston may be pushed in a considerable distance.

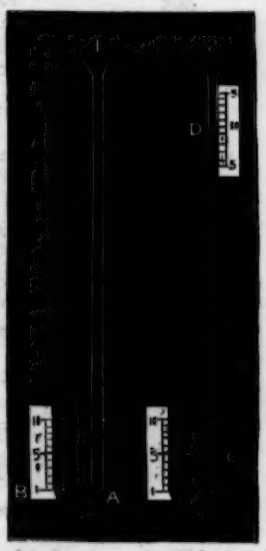


Fig. 5. Shows the mode adopted for measuring the compressibility of the air:—The apparatus is easy to construct.

Take a piece of thick glass tubing closed at one end and bent at the same end. A little mercury is poured into the bend and shaken until it stands at the same level in both arms of the tube, viz., A and B.

Now pour mercury into the long arm until the air above B is compressed into one half its original space, as shown in tube 2.

Read off the column of mercury CD. This measures the extra weight required to compress the air in the short limb into one half of its original space.

If this column of mercury be compared with the height of the barometer it will be found that the two agree. Hence the truth of statement No. 3 opposite.

Further as the column of mercury DC is lessened, the volume of air in the short limb increases. Hence the truth of statement No. 2 opposite.

4. Show this by placing a bladder half filled with air under the air pump. Upon slightly exhausting the air outside the bladder that within expands and soon fills out the bladder.

The Terms Rare and Dense.

When a given quantity (volume) of air is made to occupy a much smaller space than usual it is termed '*Dense*' on the other hand when pressure is removed and air is made to occupy a much larger space than usual the air is then said to be '*Rare*.'

These terms should be substituted for the terms *thick* and *thin* used in an earlier part of the lesson.

The property of weight and air pressure formed the subject of an entire lesson during the early part of this year's course.

BLACKBOARD SKETCH.

1. **The Existence of the Air is proved by—**
   - (*a*) Its movement in the form of wind.
   - (*b*) Its escape from a vessel under water.
2. **Extent of the Air:—**
   - (*a*) It surrounds the earth on every side.
   - (*b*) It extends to the greatest heights ever reached.
   - (*c*) Most likely it extends to from 50 to 200 miles high.
3. **Common Properties of Air:—**
   - (*a*) Air is compressible. When much compressed it is said to *dense*.
   - (*b*) Air is elastic, and has expansibility. When air is considerably expanded it is said to be *rare*.
   - (*c*) Air has weight and exerts pressure.

Quarterly Arithmetical Tests.

STANDARD I.—FIRST QUARTER.

1. 72
305
41
6
213

2. 463
120

3. 708
302

4. 7 × 2;
11 × 3;
8 × 4.

SECOND QUARTER.

1. 580
69
3
27
401

2. 516
281

3. 192
47

4. 6 × 3;
5 × 5;
7 × 6.

THIRD QUARTER.

1. 6
598
74
632
9

2. 604
8

3. 803
794

4. 12 × 4;
8 × 5;
9 × 6.

STANDARD II.—FIRST QUARTER.

1. 3,607
84
58,093
762
9,050

2. From 45,607 take 39,082.

3. 58,049 × 67.

4. 9 × 7; 12 × 9; 11 × 11.

SECOND QUARTER.

1. From 20,360 take 759.
2. 76,308 × 905.
3. 47,061 ÷ 3.
4. 65d.; 89d.; 103d.

THIRD QUARTER.

1. Take 9,807 from 10,501.
2. 39,756 × 840.
3. 83,704 ÷ 12.
4. 116d.; 137d.; 142d.

STANDARD III.—FIRST QUARTER.

1. 31,941 ÷ 13.

2. Divide two hundred and sixty-seven thousand three hundred and twenty by fifty-eight.

3. How often can nine hundred and four be subtracted from six million four hundred and twelve thousand and seventy-two?

4. The product of three numbers is 7,196,640; one number is 47, and another number is 165. What is the third number?

SECOND QUARTER.

1. If two hundred and fifty-one thousand five hundred and fifty nuts were divided equally among 249 boys and 138 girls, how many would each child receive?

2. Find the sum of five hundred and three pounds fifteen shillings and seven pence three-farthings, nine pounds twelve shillings and eight pence halfpenny, seventy-six pounds eight shillings and eleven pence farthing, nineteen shillings and ten pence, eight thousand and one pounds thirteen shillings and three-farthings, and nine hundred and twenty-eight pounds and six pence farthing.

3. Take eight hundred and seven pounds fifteen shillings and sixpence three farthings from three thousand one hundred and five pounds fourteen shillings and twopence farthing.

4. What change would be received out of a crown after paying 1s. 8d. for tea, 10½d. for coffee, 5½d. for sugar, and 9½d. for cheese?

THIRD QUARTER.

1. A book contains 196 leaves and one hundred and fifteen thousand six hundred and forty words. How many words on an average are there to a page?

2. What must be added to eight thousand nine hundred and seven pounds thirteen shillings and nine pence farthing to make ten thousand pounds?

3. What is the sum of eight thousand and fifty pounds fourteen shillings and sixpence halfpenny, ninety-seven pounds thirteen shillings and eleven pence three farthings, six hundred and four pounds eighteen shillings and fivepence, three pounds nine shillings and tenpence farthing, fifteen thousand and seventy-two pounds ten shillings and three halfpence, and nine thousand and eight pounds sixteen shillings and sevenpence three farthings?

4. A man bought a chest of oranges, containing 586, for a guinea. How much did he gain by selling them at a halfpenny each?

STANDARD IV.—FIRST QUARTER.

1. Multiply £75 18s. 10¾d. by 59.

2. Find the amount of four hundred and seven times six hundred and eighty pounds eleven shillings and eleven pence farthing.

3. What is the eleventh part of five thousand and six pounds seventeen shillings and eightpence halfpenny?

4. A farm of 592 acres is let for £1,713 2s.; find the rent of one acre.

SECOND QUARTER.

1. How many guineas are equal to two thousand and ninety-seven pounds eighteen shillings?

2. Reduce 169 tons 2 cwt. 3 lbs. 8 ozs. to ounces.

3. How many years, days, &c., are there in seventy million seconds?

4. A farmer bought a cow and 29 sheep for a hundred pounds. He paid £2 15s. 3d. for each sheep; how much did he pay for the cow?

THIRD QUARTER.

1. How many bottles, each containing 1½ pints, can be filled from 9 barrels of beer?

2. Give the difference in square yards between 38 acres 2,906 yards and 47 acres 3 roods.

3. How many dozen articles can be bought for £218 6s. at 4s. 11d. each?

4. A wheel makes 3,960 revolutions in 10½ miles; give the circumference of the wheel in feet.

STANDARD V.—FIRST QUARTER.

1. Find the value of 17,934 articles at 17s. 8¾d. each.

2. What is the rent of 8,567¾ acres at £3 16s. 9d. per acre?

3. Find the cost of 19 tons 12 cwt. 2 qrs. 23 lbs. at £4 13s. 4d. per ton.

4. What must be paid for a fence 7 miles 6 fur. 200 yds. long at £7 6s. 8d. per mile?

SECOND QUARTER.

1. If 19 gardens are made out of 16 ac. 1 ro. 3 po. at £2 16s. 8d. per acre, what is the cost of each garden?

2. Make out a bill for the following articles:—
5½ dozen books at 2s. 9d. each;
3 gross penholders at 3½d. per dozen;
7 gals. ink at 8½d. per quart;
2½ reams foolscap at 7½d. per quire.

3. If 63 men can dig a trench in 35 days, how long would it take 45 men?

4. What distance could I travel for 11s. 6d., if a journey of 286 miles cost £1 15s. 9d.?

THIRD QUARTER.

1. Simplify $\frac{1}{4} + \frac{3}{7} + \frac{5}{8} + \frac{7}{9}$.

2. If the tax on a house rented at £27 10s. be £2 10s. 5d., what would be the tax on a rental of £38 15s.?

3. From $\frac{1}{4}$ take $\frac{1}{10}$. What must be added to the result to make $\frac{3}{8}$?

4. I had a sovereign, and gave away $\frac{1}{5}$ to one person, and $\frac{2}{5}$ to another. What fraction of the sovereign had I left?

STANDARD VI.—FIRST QUARTER.

1. Simplify:—$(\frac{3}{7} + \frac{8}{21}) \div (1\frac{1}{6} - \frac{9}{10})$.

2. Divide ·3213 by 8·5, and subtract the quotient from the product of ·076 and 5·3.

3. If I went $\frac{1}{10}$ of a journey on foot, $\frac{1}{2}$ on horseback, and the remaining distance of 93 miles by rail, what was the length of the journey?

4. From $\cdot\dot{3}\dot{5}\dot{7}1428\dot{5}$ take $\cdot\dot{2}\dot{7}$, and express the result as a vulgar fraction.

SECOND QUARTER.

1. Reduce the sum of $\frac{1}{4}$ of a guinea, $\frac{1}{3}$ of a crown, and $\frac{1}{2}$ of sixpence, to the fraction of half-a-guinea.

2. If 27½ yards of cloth cost £10 13s. 1½d., what will 38¾ yards cost?

3. Find the difference between 7·34 of 6s. 3d. and 3·85 of 8s. 9d.

4. A bankrupt owes £1,826 13s. 8d., and he is able to pay 18s. 9d. in the pound. What is the value of his assets?

THIRD QUARTER.

1. Find the simple interest on £895 for nine months at 3½ per cent.

2. After paying an income-tax of 5d. in the pound, a person had £558 2s. 6d.; what was the amount of his income?

3. Find the amount of £906 12s. 6d. for 7½ years at 5¼ per cent.

4. What sum will amount to £2,356 in 6 years at 4 per cent.?

STANDARD VII.—FIRST QUARTER.

1. If it cost £15 to keep 16 horses for 5 weeks when corn is 4s. 8d. per bushel, what will it cost to keep 28 horses for 21 days when corn is 4s. per bushel?

2. In what time at 4½ per cent. per annum, will £765 amount to £885 9s. 9d.?

3. If 8 men or 15 boys can mow 16 acres in 6 days, how long would it take 18 men and 45 boys to mow 28 acres?

4. What is the compound interest on £520 for 3 years at 5 per cent.?

SECOND QUARTER.

1. In a class of 49 boys, 7 are 10 years old, 18 are 11 years, 13 are 12 years, and the remainder are 13 years. What is the average age of the class?

2. What is the discount on a bill of £323 13s. 3d. due in 2 years at 4 per cent.?

3. If 50 articles were sold at 5 for 3d., 90 at 9 for 5d., and 100 at 10 for 7d., what was the average price per dozen?

4. At what rate per cent. will £405 amount to £461 14s. in 4 years?

THIRD QUARTER.

1. A grocer mixes 19 lbs. of tea at 2s. 6d. per lb. with 7 lbs. at 3s. 4d. per lb., and sells the mixture at 2s. 10d. per lb. What percentage does he gain on his outlay?

2. In a school there are 42 per cent. boys, 39 per cent. girls, and 57 infants. How many children are there in the school?

3. A horse is sold for £60 at a *loss* of 4 per cent. At what price should it have been sold to *gain* 4 per cent?

4. In 1881 the population of a town was 42,112, and it had increased 12 per cent. since 1871. What was the population in 1871?

ANSWERS.

STANDARD I.

	FIRST.	SECOND.	THIRD.
1.	637	1,080	1,319
2.	343	235	596
3.	406	145	9
4.	14; 33; 32	18; 25; 42	48; 40; 54

STANDARD II.

	FIRST.	SECOND.
1.	71,596	19,601
2.	6,525	69,058,740
3.	3,889,283	15,687
4.	63; 108; 121	5s. 5d.; 7s. 5d.; 8s. 7d.

THIRD.

1. 694
2. 33,395,040
3. 6,975 + 4
4. 9s. 8d.; 11s. 5d.; 11s. 10d.

STANDARD III.

	FIRST.	SECOND.	THIRD.
1.	2,457	650 nuts	295 words
2.	4,608 + 56	£9,520 10s. 8½d.	£1,092 6s. 2¾d.
3.	7,093 times	£2,297 18s. 7½d.	£32,838 3s. 6¾d.
4.	928	1s. 2½d.	3s. 9d.

STANDARD IV.—FIRST.

1. £4,480 14s. 10½d.
2. £277,002 18s. 6¾d.
3. £455 3s. 5d. + 69
4. £2 17s. 10½d.

SECOND.

1. 1,998 guineas.
2. 6,060,600 ounces.
3. 2 years 80 days 4 hrs. 26 min. 40 sec.
4. £19 17s. 9d.

THIRD.

1. 1,728 bottles.
2. 44,284 square yards.
3. 74 dozen.
4. 14 feet.

STANDARD V.—FIRST.

1. £15,897 14s. 10½d.
2. £32,878 14s. 9¾d.
3. £91 12s. 7½d.
4. £57 13s. 4d.

SECOND.

1. £2 8s. 6½d.
2. £9 1s. 6d. + 10s. 6d. + 19s. 10d. + £1 11s. 3d. = £12 3s. 1d.
3. 49 days
4. 92 miles

THIRD.

1. $2\frac{9}{16}$
2. £3 11s. 0½d.
3. $\frac{5}{12}$
4. $\frac{11}{18}$

STANDARD VI.—FIRST.

1. $3\frac{1}{15}$
2. 365
3. 120 miles
4. $\frac{35}{144}$

SECOND.

1. $\frac{13}{21}$
2. £15 0s. 3¾d.
3. 12s. 2¼d.
4. £1,712 10s. 3¾d.

THIRD.

1. £23 9s. 10½d.
2. £570.
3. £1,263 12s. $2\frac{1}{19}$d.
4. £1,900.

STANDARD VII.—FIRST.

1. £13 10s.
2. 3½ years.
3. 2 days.
4. £81 19s. 3¾d.

SECOND.

1. 11¼ years.
2. £23 19s. 6d.
3. 7½d.
4. 3½ per cent.

THIRD.

1. 4 per cent.
2. 300 children.
3. £65.
4. 37,600.

—o—

Model Answers to the Matriculation Questions.

JUNE, 1887.

Geography.

1. *Name and describe the position of five leading seaports in Great Britain, and state the circumstances that have contributed to the importance of each.*

(1) The five leading seaports in Great Britain are London, Liverpool, Glasgow, Bristol, and Hull.

(*a*) *London*, with over 4,000,000 inhabitants, owes its pre-eminence as a seaport in the first place to its magnificent river and splendid docks, and secondly to the immense supplies of material for the food and trade of its inhabitants.

(*b*) *Liverpool* (population nearly 600,000) has developed during the last half century until, in the amount of its foreign trade it has outstripped London. This is mainly due to the growth of the cotton trade in the neighbouring towns. The raw cotton from America, India, and Egypt is brought chiefly to Liverpool, and from thence distributed over the manufacturing towns of South Lancashire and Cheshire. It is also the outlet for the manufactured article to all parts of the world.

(*c*) *Glasgow* (population over 500,000), on the Clyde, has developed with the growth of the cotton and iron industries in the town and neighbourhood. Shipbuilding has also developed enormously on the banks of the Clyde.

(*d*) *Bristol* (population over 200,000), on the Avon, is the outlet for the West of England trade. It has also extensive manufactures of its own, and is most favourably situated for the Irish and American trade.

(*e*) *Hull* (population nearly 200,000) is the chief outlet for the Yorkshire trade. It is most conveniently placed for trade with the Baltic and German ports.

2. *Name the rivers debouching on the coast between the Mouth of the Thames and Aberdeen, with the principal towns situate on each.*

(2)

NAME OF RIVER.	PRINCIPAL TOWNS.
Chelmer	Chelmsford
Colne	Colchester
Stour	Harwich
Orwell	Ipswich
Yare and Wensum	Yarmouth, Norwich
Great Ouse	King's Lynn, Ely, Cambridge, Huntingdon and Bedford
Nen	Peterborough, Northampton
Welland	Spalding
Witham	Boston, Lincoln
Humber	Grimsby, Hull
Trent	Gainsborough, Newark, Nottingham, Burton
Yorkshire Ouse	Goole, Selby, York
Tees	Middlesborough, Stockton, Darlington (near)
Wear	Durham, Sunderland
Tyne	Tynemouth, South Shields, Newcastle
Tweed	Berwick
Forth	Leith, Stirling
Tay	Dundee, Perth
North and S. Esk	Montrose

3. *Describe the different routes to India, and point out the advantages and disadvantages of each so far as England is concerned.*

(3) (*a*) The *sea route* viâ the Cape of Good Hope. This has the advantage of open sea the entire distance but has the disadvantage of being the longest journey.

(*b*) The *overland route* :—Of these there are three to select from

1. By ship the entire journey, viâ Gibraltar, the Mediterranean, Suez Canal, Red Sea, and Arabian Sea.
2. Across France to Marseilles, and thence by the Mediterranean as before.
3. Across France into Northern Italy to Brindisi, thence ship down the Adriatic into the Mediterranean, Suez Canal, &c.

The last route is the most expeditious of the three. It shares a disadvantage with the second route in the necessity for transshipment, and all three routes might in case of war be entirely closed to Englishmen. The Gibraltar route avoids trans-shipment. It might, however, be risky if we were at war, or if Egypt were in an unsettled state.

4. *Show upon an outline map the different European settlements in Africa, as well as the places on that continent now under occupation by different European States.*

(4) The following are the European possessions in Africa. They form the material required for answering the question.

DISTRICTS IN AFRICA.	EUROPEAN COUNTRY TO WHICH THEY EITHER ENTIRELY OR IN PART BELONG
1. Algeria	France
2. Tunis	Turkey
3. Eastern Tripoli	Turkey
4. Egypt	Turkey, England (Protectorate)
5. Massowah	Italy
6. Senegambia	
(*a*) Senegal River and along the coast to near Gambia; **St. Louis**	France
(*b*) Geba River & San Domingo River; **Bissao**	Portugal
(*c*) Gambia; Bathurst, Sierra Leone, Sherboro Island	England
7. Gold Coast; **Cape Coast Castle**	England
8. Fernando Po, Annabon	Spain
9. Princes Island, St. Thomas	Portugal
10. Inlet of Gaboon	France
11. West Africa from just north of Congo mouth to Cape Frio	Portugal
12. Cape Colony, Natal, the Transvaal (suzerainty in foreign affairs only) Bechuanaland (Protectorate)	England
13. Eastern Africa from Cape Delgado to Delagoa Bay; **Quilimane**	Portugal
14. Ascension Islands, St. Helena *on the East*, Socotra and Perim *on the West*	

History.

1. *State the circumstances connected with the fall of the Roman supremacy in Britain. What traces did that supremacy leave?*

(1) Throughout the entire Roman Empire the population was declining as the landed proprietors widened their possessions, and as the sturdy cultivators of the soil gradually sunk into a state of indifferent serfdom. Heavy taxation necessary to maintain the army and the central government crushed alike both town and country. Above all the despotic system of Roman Government failed to stimulate local independence and life. At this time the Goths, who had already conquered Spain, were threatening Italy, and it was to defend Italy against these barbarians that the Roman legions were withdrawn from Britain.

The chief traces of the Roman supremacy which remained after the departure of the Roman legions were to be found in habits of life and in the language of the few important towns, elsewhere the Roman influence appears to have completely died out. For the most part the Roman law, its literature, its manners, and its faith, went with the departure of its legions from our shores.

2. *Describe (giving the dates, titles, and substance of the chief enactments) the course of the constitutional and ecclesiastical struggles of the reign of Henry II.*

(2) (*a*) *Constitutional struggles* :—Henry II. in successive assizes commenced the fabric of our present judicial legislation. The first was named the *Clarendon assize*, 1166. In the provisions of this assize we find the origin of trial by jury as well as the functions of our present 'Grand jury.' At the *Northampton assize*, 1176, Henry divided the Kingdom into six circuits, to each of which he assigned three itinerant judges. In 1178 five judges were set apart to hear all suits that were brought before the king, all the business in fact which came at a later period before the Courts of Queen's Bench, Exchequer, and Common Pleas.

(*b*) *Ecclesiastical Struggles.*—The Church Courts, at the time of Henry's accession held the exclusive right of exercising justice over the whole body of educated men throughout the realm. Henry at once proposed to the bishops that a clerk convicted of a crime should be deprived of his orders and handed over to the civil tribunals. In this struggle against the power of the Church, Henry trusted that Becket would assist him. His favourite, however, instead of assisting the King, met his proposals with the most strenuous opposition. The Constitutions of Clarendon, 1164, were accepted by the bishops and finally by the primate. The chief articles are :—

1. The prelate elect to do homage to the King for his lands before consecration.
2. Clerks charged with crimes to be tried by the King's Courts.
3. No bishop to quit the realm without the consent of the King.
4. No royal servant or tenant in chief to be excommunicated without the King's consent.

The murder of Becket, 1170, weakened the application of these articles for a time, but in the end the victory remained with the King. Church appointments were practically in his hands, and the King's Court controlled those of the church.

3. *Discuss the claim of Edward I. to suzerainty in Scotland: and describe the position of Bruce in relation to the English crown when the struggle began.*

Edward I. claim to suzerainty in Scotland.

(*a*) Strath-clyde, which had been conquered by the English King, was granted to Malcolm of Scotland on the condition of military service to the English King. Similarly the Lothians were afterwards held by the Scottish King.

(*b*) The most direct claim to over-lordship by the English monarch rested upon the capture of William the Lion by the English during the reign of Henry II. To gain his freedom William consented to hold his crown under suzerainty of Henry. Richard, however, allowed Scotland to repurchase her freedom, and afterwards the Scotch king paid homage to the English monarch on account of the lands he had acquired within the English realm.

(*c*) The death of Alexander III. of Scotland left the succession to the 'Maid of Norway,' and her early death, together with the separate claims of some thirteen aspirants to the Scotch throne, many of whom appealed directly to Edward, brought Edward's influence into greater prominence, and at the Parliament at Norham, 1291, the Scotch lords formally admitted Edward's direct suzerainty.

(*d*) Originally the Bruce family formed a part of the Yorkshire baronage. Through marriage they became connected with the Scotch nobility. During the struggle between Edward and Balliol and Wallace, Robert Bruce was on the English side. He had in fact been trained in the English court, and stood high in the King's favour.

4. *Give a short account of the careers of Wolsey and Thomas Cromwell, and contrast their relations to the chief movements of the time.*

(4) *Wolsey*, son of a townsman of Ipswich, entered the Church, became Royal chaplain, and afterwards a civil servant of the crown. His power increased until he became a minister, and finally the chief minister of Henry VIII. His aim was to make the king supreme. He lessened the power of the prelates and nobles who sat in council with the king. Authority in home and foreign affairs was concentrated in the hands of a single minister, and that minister was Wolsey. His abilities were such that single-handed he was able to do the whole work with a success that far exceeded all expectation. This concentration of secular power in a single hand accustomed England to the personal government which began with Henry VIII. Further, Wolsey's sole control of the Church, and the consequent cessation of appeals to Rome, prepared the way for men to acquiesce in Henry's supremacy in religious affairs. Wolsey was stoutly opposed by the Commons. Their services were largely dispensed with however by the new mode of raising money which he devised. His career was as remarkable as it was short. From conscientious scruples he failed to support Henry in his marriage schemes, and thenceforth he rapidly lost the King's favour, was arrested for high treason, and died at Leicester whilst on his way to the Tower.

Thomas Cromwell, like Wolsey, rose from obscurity to more than kingly supremacy. At Wolsey's fall, Cromwell stood manfully by the old Cardinal. After his fall he advised the king to appeal no more to Rome for a divorce, but to boldly assume supremacy in church matters and obtain a divorce from his own Ecclesiastical Courts. Wolsey maintained the connection of the Romish Church, and became the Papal legate. Cromwell sought to make the Church of England entirely distinct from that of Rome, and enforced a heavy penalty upon the Clergy after Wolsey's fall, on account of the exercise of Papal power which they had sought and allowed. The Clergy obtained forgiveness by the payment of £1,000,000 present value and the acknowledgment of the king as 'Protector and Supreme head of the Church and Clergy of England.'

Whilst Wolsey held high the position of the Church, Cromwell now sought to make it completely subservient to the State; and the Act of Supremacy, 1534, gave him the complete control he desired, and which he immediately began to exercise.

Wolsey certainly strained the law, especially in not assembling Parliament, at the same time he religiously observed the freedom of justice. Cromwell, on the other hand, did not shrink from influencing both judges and juries, and he fearlessly used Parliament for the accomplishment of his purpose. Wolsey was supported by the Church. Cromwell was not, and when he failed in a foreign alliance, the king turned against his powerful minister; the entire force of the Church was also brought to bear against him, and Norfolk, who represented the position left by Wolsey, was commissioned to arrest him.

5. *Name the successive Ministries under Charles II., and show what line of policy was represented by each.*

(5) The Convention Parliament was composed of men who were for the most part Presbyterian, though loyal, and strongly adverse to any form of despotism. It was followed the same year by a Parliament under Clarendon, which was strongly Royalist, and which readily restored some prerogative to the crown. Acts were passed against the Puritans, notably the Act of Uniformity, which enforced the use of the Prayer Book, and the *Five Mile Act*, which forbade ministers who had not subscribed to the Act of Uniformity to teach in schools, or to settle within five miles of any corporate town.

In 1667 Clarendon fell into disgrace, and with his dismissal the Cavalier party in the council ceased to exist, and the old Presbyterian Party, under Ashley, came to the front. Charles now proposed toleration, largely however in favour of the Catholics. This was still further apparent when the Declaration of Indulgence was proclaimed. With the passing of the Test Act, which required all persons holding office to subscribe to the rites of the Church of England, the Cabal Ministry broke up, and the Earl of Danby became leading minister, with Shaftesbury as his chief opponent. The Popish tendencies of the King were manifest by the attempt of Danby to pass a bill making all in office declare on oath that 'they consider resistance to the King unlawful, and that they will make no alteration in Church and State.' This was rejected by the Commons, and soon after an Act was passed disabling Papists from sitting in either House of Parliament. The following year Danby was impeached and committed to the Tower, and Shaftesbury returned. His object was to ensure the Protestant succession, and in this he was supported by Parliament. The Exclusion Bill was the result. The King dissolved his fourth Parliament, and summoned the fifth to meet at Oxford with the same result. Henceforth Charles determined to rule without a Parliament, and the Duke of York became his chief adviser.

6. *Trace the foreign policy of James I., and contrast it with that of his predecessor.*

(6) James I. during the life time of Cecil, continued the foreign policy of his predecessor. Afterwards, however, he proceeded to undo all that the struggle of Elizabeth and triumph over the Spanish Armada had done. Instead of allying himself with France against Spain, he began a series of negotiations for the marriage of his son with a Princess of Spain. A second struggle between the forces of Protestantism and Romanism was gathering on the continent of Europe. Elizabeth had nobly stood by the Protestant cause, but James, whilst openly favouring the cause of German Protestantism was secretly favouring the Spanish cause by granting a license for the export of arms to Spain. In 1623, he caused the English fleet which had been cruising by way of menace off the coast of Spain to be recalled, and dismissed the ministers who opposed an alliance with Spain. Parliament, however, stood firm, and the failure of the marriage alliance with Spain enabled the fourth parliament to declare war with that country, and immediately a treaty of marriage for Prince Charles was arranged with France. A return to the policy pursued by Elizabeth seemed now secured, and the death of the King, in 1625, gave it further assurance.

Arithmetic and Algebra.

1. Find to six decimals the difference between the cube of 4·791288 and the square of 10·487655.

Contracted multiplication to eight places of decimals :—

```
      4·79128800
         8821974
     1916515200
      335390160
       43121592
         479129
          95826
          38330
           3833
     22·95644070
```

Contracted multiplication to seven places of decimals :—

```
     22·9564407
         8821974
      918257628
      160695085
       20660797
         229564
          45913
          18365
           1836
     109·9009188
```

Contracted multiplication to seven places of decimals :—

10·48765500
55678401
10487655
4195062
839012
73413
6292
524
109·9909074

109·9909188
109·9909074
·0000114

∴ Difference = ·000011. Ans.

2. Calculate to five decimals the value of $\sqrt{\left(\frac{\sqrt{13}+3}{\sqrt{13}-3}\right)}$.

$$\frac{\sqrt{13}+3}{\sqrt{13}-3} = \frac{(\sqrt{13}+3)(\sqrt{13}+3)}{(\sqrt{13}-3)(\sqrt{13}+3)}$$
$$= \frac{13+6\sqrt{13}+9}{13-9}$$
$$= \frac{22+6\sqrt{13}}{4}$$
$$= \frac{11+3\sqrt{13}}{2}$$
$$= \frac{11+\sqrt{117}}{2}$$

$\sqrt{117}$ (10·8166
100
208 | 1700
1664
2161 | 3600
2161
21626 | 143900
129756
216326 | 1414400
1297956
116444

We can obtain four (or five) more decimals by ordinary division :—

108166
2
216332) 1164440 (5382
1081660
827800
648996
1788040
1730656
573840
432664
141176

∴ $\sqrt{117}$ = 10·81665382...

10·81665382
11
2 | 21·81665382
10·90832691 (3·302
9
63 | 190
189
6602 | 18326
13204
6604)512291 (77
46228
50011
46228
3783

= 3·30277. Ans.

3. Determine the longest paying voyage, when freight is a penny a mile for 10 tons, of a steamer carrying 2,000 tons of coal and cargo; supposing the steamer to go 10 knots (miles) an hour with a consumption of 60 tons of coal a day, the coal costing 12s. a ton, and the steamer £20 a day for wages, repairs, and interest on capital.

Daily expenses = (12s. × 60) + £20
= £36 + £20
= £56;

Daily charge for 10 tons = (10 × 24)d.
= £1;

∴ Cargo must exceed 560 tons to make profit;

∴ Voyage must be less than $\frac{2000-560}{60}$ days

$= \frac{1440}{60}$ days

= 24 days.

Difference of a day's voyage gives a difference of £18, or £6, in the daily profit.

Profit on a voyage of 23 days = £6 × 23 = £138;
,, ,, ,, 22 ,, = £12 × 22 = £264;
,, ,, ,, 21 ,, = £18 × 21 = £378;
&c., &c.,
,, ,, ,, 13 ,, = £66 × 13 = £858;
,, ,, ,, 12 ,, = £72 × 12 = £864;
,, ,, ,, 11 ,, = £78 × 11 = £858

It is thus evident that the profit increases to the twelfth day, and then it decreases.

∴ Longest paying voyage = 12 days. Ans.

4. Prove that if $\frac{(a-b)(c-d)}{(a-c)(b-d)} = x$, then

(i.) $\frac{1}{x}$ is the resulting expression from an interchange of a, d, or b, c;

(ii.) $1-x$ from an interchange of b, d, or a, c;

(iii.) $\frac{x}{x-1}$ from an interchange of c, d, or a, b; but that no alteration takes place in the expression.

$$\frac{(1-x+x^2)^3}{x^2(1-x)^2}.$$

If $x = \frac{(a-b)(c-d)}{(a-c)(b-d)}$;

(i.) $\frac{1}{x} = \frac{1}{\frac{(a-b)(c-d)}{(a-c)(b-d)}}$
$= \frac{(a-c)(b-d)}{(a-b)(c-d)}$
= interchange of b, c;
or, $\frac{1}{x} = \frac{(b-d)(a-c)}{(c-d)(a-b)}$
$= \frac{(d-b)(c-a)}{(d-c)(b-a)}$
= interchange of a, d.

(ii.) $1-x = 1 - \frac{(a-b)(c-d)}{(a-c)(b-d)}$
$= \frac{(a-c)(b-d)-(a-b)(c-d)}{(a-c)(b-d)}$
$= \frac{ab-bc-ad+cd-ac+bc+ad-bd}{(a-c)(b-d)}$
$= \frac{ab+cd-ac-bd}{(a-c)(b-d)}$
$= \frac{a(b-c)-d(b-c)}{(a-c)(b-d)}$
$= \frac{(a-d)(b-c)}{(a-c)(b-d)}$
$= \frac{(a-d)(c-b)}{(a-c)(d-b)}$
= interchange of b, d;

$$\text{or, } 1 - x = \frac{(c-b)(a-d)}{(c-a)(b-d)}$$

= interchange of a, c.

$$\text{(iii.)} \quad \frac{x}{x-1} = \frac{\dfrac{(a-b)(c-d)}{(a-c)(b-d)}}{\dfrac{(a-b)(c-d)}{(a-c)(b-d)} - 1}$$

$$= \frac{(a-b)(c-d)}{(a-b)(c-d) - (a-c)(b-d)}$$

$$= \frac{(a-b)(c-d)}{ac - bc - ad + bd - ab + bc + ad - cd}$$

$$= \frac{(a-b)(c-d)}{ac + bd - ab - cd}$$

$$= \frac{(a-b)(c-d)}{a(c-b) - d(c-b)}$$

$$= \frac{(a-b)(c-d)}{(a-d)(c-b)}$$

$$= \frac{(a-b)(d-c)}{(a-d)(b-c)}$$

= interchange of c, d;

$$\text{or, } \frac{x}{x-1} = \frac{(b-a)(c-d)}{(b-c)(a-d)}$$

= interchange of a, b.

If for x we write $\frac{1}{x}$,

Then, $\dfrac{(1-x+x^2)^3}{x^3(1-x)^3}$ becomes $\dfrac{\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^3}{\frac{1}{x^3}\left(1-\frac{1}{x}\right)^3}$,

$$\text{or } \frac{\left(\frac{x^2-x+1}{x^2}\right)^3}{\frac{1}{x^3}\left(\frac{x-1}{x}\right)^3},$$

$$\text{or } \frac{(1-x+x^2)^3}{x^3(1-x)^3};$$

Again, if for x we write $1-x$,

Then, $\dfrac{(1-x+x^2)^3}{x^3(1-x)^3}$ becomes $\dfrac{[1-(1-x)+(1-x)^2]^3}{(1-x)^3[1-(1-x)]^3}$,

$$\text{or } \frac{(1-x+x^2)^3}{x^3(1-x)^3};$$

Lastly, if for x we write $\frac{x}{x-1}$,

Then, $\dfrac{(1-x+x^2)^3}{x^3(1-x)^3}$ becomes $\dfrac{\left[1-\frac{x}{x-1}+\frac{x^2}{(x-1)^2}\right]^3}{\left(\frac{x}{x-1}\right)^3\left(1-\frac{x}{x-1}\right)^3}$

$$\text{or } \frac{(1-x+x^2)^3}{x^3(1-x)^3};$$

but these are the only possible results after a, b, c, d have been exchanged;

∴ In the expression $\dfrac{(1-x+x^2)^3}{x^3(1-x)^3}$ no alteration takes place.

5. Prove that

$$(x^2+14x+1)^3 - (x^3-33x^2-33x+1)^3 = 108x(x-1)^4.$$

$$(x^2+14x+1)^3 - (x^3-33x^2-33x+1)^3$$
$$= [(x^2+1)+14x]^3 - [(x^3+1)-33x(x+1)]^3$$
$$= (x^2+1)^3 + 3.\,14x(x^2+1)^2 + 3.\,(14x)^2(x+1) + (14x)^3$$
$$- \left\{(x^3+1)^3 - 66x(x+1)(x^3+1) + [33x(x+1)]^3\right\}$$
$$= x^6 + 3x^4 + 3x^2 + 1 + 42x(x^4+2x^2+1)$$
$$+ 588x^2(x^2+1) + 2744x^3$$
$$- [x^6 + 2x^3 + 1 - 66x(x^4+x^3+x+1)$$
$$+ 1089x^2(x^2+2x+1)]$$
$$= x^6 + 3x^4 + 3x^2 + 1 + 42x^5 + 84x^3 + 42x$$
$$+ 588x^4 + 588x^2 + 2744x^3$$
$$- (x^6 + 2x^3 + 1 - 66x^5 - 66x^4 - 66x^2 - 66x$$
$$+ 1089x^4 + 2178x^2 + 1089x^2)$$
$$= x^6 + 42x^5 + 591x^4 + 2828x^3 + 591x^2$$
$$+ 42x + 1 - x^6 + 66x^5 - 1023x^4$$
$$- 2180x^3 - 1023x^2 + 66x - 1$$
$$= 108x^5 - 432x^4 + 648x^3 - 432x^2 + 108x$$
$$= 108x(x^4 - 4x^3 + 6x^2 - 4x + 1)$$
$$= 108x(x-1)^4.$$

6. Simplify

(i.) $\dfrac{x+8}{(x-15)(x-17)} + \dfrac{x+15}{(x-8)(x-17)} - \dfrac{x+17}{(x-8)(x-15)}$;

(ii.) $\dfrac{x^4 - 8x^2y^2 + 16y^4}{x^3 - 6x^2y + 12xy^2 - 8y^3}$

(i.) $\dfrac{x+8}{(x-15)(x-17)} + \dfrac{x+15}{(x-8)(x-17)} - \dfrac{x+17}{(x-8)(x-15)}$

$$= \frac{(x+8)(x-8) + (x+15)(x-15) - (x+17)(x-17)}{(x-8)(x-15)(x-17)}$$

$$= \frac{x^2 - 64 + x^2 - 225 - x^2 + 289}{(x-8)(x-15)(x-17)}$$

$$= \frac{x^2}{(x-8)(x-15)(x-17)}. \text{ Ans.}$$

(ii.) $\dfrac{x^4 - 8x^2y^2 + 16y^4}{x^3 - 6x^2y + 12xy^2 - 8y^3}$

$$= \frac{(x^2-4y^2)^2}{(x-2y)^3}$$

$$= \frac{[(x+2y)(x-2y)]^2}{(x-2y)^3}$$

$$= \frac{(x+2y)^2}{x-2y}. \text{ Ans.}$$

7. Find the sum of all the numbers from one to a thousand, which are not divisible by 2 or 5.

Sum = (sum of odd numbers—sum of odd numbers divisible by 5) from 1 to 1,000.

= (1 + 3 + 5 +, &c., to $\frac{1000}{2}$ terms).

− (5 + 15 + 25 +, &c., to $\frac{1000}{10}$ terms).

$$= \left\{(1+999) \times \frac{500}{2}\right\} - \left\{(5+995) \times \frac{100}{2}\right\}$$
$$= (1000 \times 250) - (1000 \times 50)$$
$$= 1000 \times 200$$
$$= \underline{200{,}000}. \text{ Ans.}$$

8. Find the sum of a given number of terms of a geometrical progression, given the first term and the common ratio.

Reduce to a fraction in its simplest form $\dot{0}12345679\dot{0}$.

Let a denote the first term, r the common ratio, n the number of terms, and s the sum of the terms.

Then,

$$s = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1};$$
$$\therefore sr = \quad ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n;$$

Hence, by subtraction,

$$sr - s = ar^n - a,$$
$$s(r-1) = a(r^n - 1),$$
$$\therefore s = \underline{\frac{a(r^n-1)}{r-1}}.$$

Recurring decimals are cases of infinite geometrical progressions.

$$\dot{0}12345679\dot{0} = \frac{12345679}{999999999}$$

12345679)999999999(81
98765432
12345679
12345679

$$= \underline{\tfrac{1}{81}}. \text{ Ans.}$$

9. Solve the equations :—

(i.) $(3x - 8)(3x + 2) - (4x - 11)(2x + 1) = (x - 3)(x + 7)$;

(ii.) $2x + 3y = 5$, and $3x + 5y = 8$.

(i.) $3x - 8)(x + 2) - (4x - 11)(2x + 1) = (x - 3)(x + 7)$

Simplifying] $9x^2 - 18x - 16 - (8x^2 - 18x - 11) = x^2 + 4x - 21$

Removing brackets] $9x^2 - 18x - 16 - 8x^2 + 18x + 11 = x^2 + 4x - 21$

Collecting] $x^2 - 5 = x^2 + 4x - 21$

Subtracting x^2 from each side] $-5 = 4x - 21$

Transposing] $4x = 21 - 5$

$= 16$

Dividing each side by 4] $\therefore$ $\underline{x = 4}$

(ii.)

(1) $2x + 3y = 5$
(2) $3x + 5y = 8$

(1) $\times 5$ → (1) $10x + 15y = 25$
(2) $\times 3$ → (2) $9x + 15y = 24$

Subtracting: $x = 1$.

(1) $2x + 3y = 5$

Substituting value of x] $2 + 3y = 5$

Transposing] $3y = 5 - 2$

Dividing each side by 3] $\therefore$ $\underline{y = 1}$.

$\therefore$ $\underline{x = 1 \text{ and } y = 1}$. Ans.

10. With $4\frac{3}{4}$ hours at his disposal, how far can a man go out by train at 20 miles an hour, in order to walk back at $3\frac{3}{4}$ miles an hour?

Let x = distance in miles;

Then $\frac{x}{20}$ = time by rail in hours,

And $\frac{x}{3\frac{3}{4}}$ = time on foot in hours;

$\therefore \frac{x}{20} + \frac{x}{3\frac{3}{4}} = 4\frac{3}{4}$

$\frac{x}{20} + \frac{4x}{15} = \frac{19}{4}$

L. C. M. $= 60$] $3x + 16x = 19 \times 15$

$19x = 19 \times 15$

$\therefore x = 15$

$\therefore$ Distance = $\underline{15 \text{ miles}}$. Ans.

The remainder of answers will appear in our next issue.

Science Notes.

IT is encouraging to notice that vacation courses in Natural Science are now being conducted at Edinburgh. There was a time when the only higher scientific education within the reach of the ordinary teacher was the summer courses at South Kensington. We question whether South Kensington has received anything like the appreciation and gratitude for instituting these courses it has deserved. There can be no doubt however that it has done a great work, the fruit of which is only beginning to appear. We trust the summer courses at South Kensington may continue to be as popular as ever they were, and as stimulating and instructive. But it is gratifying to know that the central institution of the City Guilds and the University of Edinburgh have seen fit to organize similar courses. Would it not be well if Oxford and Cambridge would follow suit? A course of lectures and laboratory work, extending over six weeks, open to all who gave evidence of competence to profit thereby, would be hailed as a boon by many engaged in teaching. Science is constantly advancing, and men who love their work and wish to keep themselves abreast of the time would gladly spend some, or even the whole, of their holiday in deepening their own knowledge and learning improved methods. We believe there is a genuine desire for higher instruction as well as for the newest and best methods of presenting what is simple and elementary. Why should a student be excluded from the finest institutions in the country because he cannot become a resident for three or four years? This is not so abroad. The universities of Oxford and Cambridge could do nothing ing more useful or more popular than organise next year a four or six weeks' course of advanced instruction in science, with special regard to the wants of teachers in elementary and middle class schools.

* * *

A NATIONAL association for the promotion of technical education has now been formed. Lord Hartington has taken a prominent part in its proceedings, and at a meeting held on the 1st he made an important speech. After referring to our past commercial supremacy, and giving some of the reasons for it, he pointed out that other nations had been quick to perceive that wonderful scientific advances had been made, and more and more science was being applied to the occupations of the world. The nations were quick to perceive this, and were now striving to make their position equal to ours by developing at immense cost to the state and public funds that scientific instruction which would enable their manufactories and workmen to compete successfully with ours. If we were passive in the matter, if we were indolent, it was conceivable not only that foreign nations would rival us, but that they might also succeed in passing us, with consequences which it would be difficult to contemplate. If we were satisfied to go on as we were, if we are content to rely in the future, as in the past, on those advantages which had given us our present position, if we did not think it necessary to organize more completely our system of technical instruction than at present, that decision should be the result of deliberate and well-formed consideration and not the result of apathy or indolence. The Government are evidently fully alive to the necessity of dealing with this subject without delay, and a Bill is expected to be introduced upon the subject within a few days.

* * *

THE Recreative Evening Schools' Association are organizing a system of elementary instruction in many branches of natural science for the coming winter session. It is expected that there will be at least one hundred of the Board Schools employed as centres during the coming session (October to April).

It is worth while to consider how far such classes could be conducted in other large towns in the country. The work of the teacher is often that of a missionary. He has to create a desire for the knowledge he has come to impart. It is in these recreative classes that interest can be stimulated, and curiosity aroused. We must be content to make haste slowly. Children have often been disgusted with the small amount of natural science that they have been taught. It has been a drudgery in some cases. They have, wrongly no doubt, conceived a dislike to such instruction, and they need to be cleverly handled if they are to work seriously at the subject again.

THE delegates of the Clarendon Press have undertaken to publish a new botanical magazine, entitled 'Annals of Botany.' It will be edited by Prof. Baley Balfour, Dr. Vines and Prof. W. G. Farlow. The papers will be adequately illustrated, and will relate to all the branches of botanical science. There will also be articles on the history of Botany, reviews and criticisms of books, short notes and letters. An attempt will be made to bring promptly under the notice of the readers of the magazine all the newest discoveries. Secretaries of institutions are requested to communicate as early as they can respecting the publication of papers relating to botany in any of its branches.

Mental Arithmetic, its Principles and Practice.

BY REV. A. D. CAPEL, M.A.,

Author of 'Catch Questions in Arithmetic,' 'Tips in Algebra,' 'Easy Steps in Trigonometry,' etc., etc.

ANSWERS TO EXERCISES.

CHAPTER IX.

1. $9\frac{2}{3}$.
2. $6\frac{1}{10}$.
3. $46\frac{1}{4}$.
4. $69\frac{1}{10}$.
5. $603\frac{1}{8}$.
6. $353\frac{1}{8}$.
7. $1000\frac{1}{12}$.
8. $6493\frac{1}{10}$.
9. $1365\frac{1}{12}$.
10. $20,186\frac{1}{14}$.
11. $43,239\frac{1}{16}$.
12. $87,621\frac{1}{12}$.
13. $4\frac{1}{125}$.
14. $8\frac{1}{125}$.
15. $47\frac{1}{125}$.
16. $82\frac{1}{125}$.
17. $98\frac{1}{125}$.
18. $231\frac{1}{125}$.
19. $865\frac{1}{125}$.
20. $730\frac{1}{125}$.
21. $925\frac{1}{125}$.
22. $7,446\frac{63}{125}$.
23. $1,441\frac{108}{125}$.
24. $65,936\frac{118}{125}$.
25. $8\frac{20}{101}$.
26. $9\frac{46}{101}$.
27. $12\frac{20}{101}$.
28. $49\frac{37}{101}$.
29. $961\frac{1}{101}$.
30. 984.
31. $3,869\frac{44}{101}$.
32. $4,928\frac{66}{101}$.
33. $4,952\frac{20}{101}$.
34. $9,900\frac{99}{101}$.
35. $89,918\frac{1}{101}$.
36. $888,174\frac{17}{101}$.
37. $2\frac{456}{1001}$.
38. $8\frac{208}{1001}$.
39. $15\frac{455}{1001}$.
40. $78\frac{848}{1001}$.
41. $35\frac{168}{1001}$.
42. $899\frac{303}{1001}$.
43. 500.
44. 728.
45. $5465\frac{342}{1001}$.
46. 7370.
47. $69,800\frac{504}{1001}$.
48. $390,673\frac{440}{1001}$.
49. $12,346\frac{888}{1001}$.
50. $98,755\frac{1440}{10001}$.

CHAPTER X.

1. 23.
2. 37.
3. 49.
4. 57.
5. 97.
6. 121.
7. 342.
8. 478.
9. 589.
10. 623.
11. 687.
12. 516.
13. 684.
14. 1,019.
15. 2,035.
16. 2·21....
17. 2·52....
18. 2·84....
19. 3·47....
20. 3·79....
21. 924.
22. 2,310.
23. 6,048.
24. 2,695.
25. 8,008.
26. 616.
27. 504.
28. 2,310.
29. 3,080.
30. 876.
31. 23.
32. 47.
33. 97.
34. 121.
35. 247.
36. 394.
37. 489.
38. 376.
39. 482.
40. 926.
41. 10.
42. 255.
43. 66.
44. 9,025.
45. 55,225.
46. $\frac{1}{2}$.
47. $\frac{11}{12}$.
48. 13,575.
49. $\frac{11}{14}$.
50. $\frac{1}{3}$.

CHAPTER XI.

1. £63 17s. 6d.
2. £83 11s. 11d.
3. £41 1s. 3d.
4. £68 8s. 9d.
5. £107 19s. 7d.
6. £120 2s. 11d.
7. £40 0s. 6½d.
8. £5 14s. 0½d.
9. £84 0s. 6¼d.
10. £215 11s. 6¾d.
11. 18s. 9d.
12. £3 2s. 6d.
13. £6 13s. 4d.
14. £12 1s. 8d.
15. £43 8s. 9d.
16. £23 4s. 7d.
17. £29 13s. 9d.
18. £33 10s. 10d.
19. £85 16s. 8d.
20. £95 10s. 5d.
21. £44 12s. 6d.
22. £55 5s. 4½d.
23. £66 5s. 6d.
24. £16 1s. 3¾d.
25. £37 5s. 2½d.
26. £42 15s.
27. £43 11s. 7½d.
28. £67 6s. 7½d.
29. £56 16s. 0¾d.
30. £14 10s.
31. £20 16s. 6d.
32. £25 9s. 2½d.
33. £43 10s.
34. £50 14s. 2½d.
35. £47 11s. 0½d.
36. £49 17s. 6d.
37. £71 1s. 0½d.
38. 18s. 2½d.
39. £28 2s. 11d.
40. £32 4s. 7½d.
41. £59 8s.
42. £78 2s. 4½d.
43. £14 17s. 6d.
44. £20 8s.
45. £2,750.
46. £3,561 5s.
47. £12 15s.
48. £85 19s. 6d.
49. £99 15s.
50. £82 15s. 4½d.

CHAPTER XII.

SIMPLE PRACTICE.

113. In the last chapter we showed how the number of articles could be split up into terms which were multiples of 20, or 12, or 48, &c.; in this chapter we show how the principles of practice, as used in slate arithmetic, can be applied to mental arithmetic.

114. The student ought to know thoroughly the aliquot parts of £1 and 1s., &c.

115. Since 1s. 8d. or 20d. is $£\frac{1}{12}$

$£2$ 1s. 8d. $= £2\frac{1}{12} = £\frac{25}{12}$

It would therefore be very easy to write down the price of any multiple of 12 articles at £2 1s. 8d. each.

e.g., taking 72, the ans. would be £150.

116. In such a question as 86,032 articles at 17s. 6d.—since 17s. 6d. is £1 less 2s. 6d., or £1 less $£\frac{1}{8}$—we should subtract $\frac{1}{8}$ of 86,032 from itself and call them pounds thus: £(86,032 − 10,754) or £75,278.

117. In such a question as 72,819 articles at 12s. 6d., we could either add $\frac{1}{2}$ of 72,819 to $\frac{1}{8}$ of 72,819 and call the result pounds, or reduce the 12s. 6d. to $\frac{25}{2}$ shillings, or $\frac{100}{8}$ shillings, or $£\frac{10}{16}$. In other words, add on a cypher and divide by 16 thus: £45,511 17s. 6d.

118. It is always worth while to reduce the price, if it include pounds, to a mixed number, as in the Government papers we frequently find the mixed number to be one which reduces to an improper fraction, by which it is easy to multiply.

7,548 articles at £16 13s. 4d.

£16 13s. 4d. $= £16\frac{2}{3} = £\frac{50}{3} = £\frac{100}{6}$.

hence all we have to do is to imagine two cyphers at the end of 7,548 and divide by 6, which gives us £125,800.

119. In such a case as 4030 articles at £32 4s. each—since £32 4s. = 32¼ or $\frac{161}{5}$ —it reduces itself to multiplying 806 by 161, which can be done by methods shown in former chapters, giving as our result £129,766. Ans.

120. Since 19s. 7d. = 240d. − 5d. or $£\left(1-\frac{1}{48}\right)$ to find the price of 145,731 articles at 19s. 7d. each, we must subtract $\frac{1}{48}$ of 145,731, or £3,036 1s. 3d. from £145,731, which gives us £142,694 18s. 9d.

121. Since 9s. 6d. = 10s. less 6d., or £½ less ½s., to find the value of articles at 9s. 6d. each, we can take half the number and call them pounds, and from this number of pounds take the same number of shillings.

E. g., what is the value of 275 articles at 9s. 6d. each: the mental operation is £137 10s. less 137½s., or £6 17s. 6d., which gives us as our result £130 12s. 6d.

122. The value of a number of articles at some aliquot part of 100 pence—as 2s. 1d.—can be found thus:—

475 articles at 2s. 1d. each = 475 articles at 25 pence, which is 475 articles at $\frac{100}{4}$ pence, or at $\frac{100}{48}$ shillings, or at $£\frac{10}{96}$ each ∴ the result is $£\frac{4750}{96}$, or £49 9s. 7d. The division by 96 is not hard. Since 96 is within 4 of a hundred, when we divide 475 by 96, the remainder is 75 + (4 × 4) or 91, and when we divide 910 by 96 the remainder is 10 + (9 × 4) or 46, and this reduced to 920 gives us 9 with a remainder of 20 + (4 × 9) or 56, and this reduced to pence and divided by 96 is simply dividing 56 by 8 which gives us the 7d.

123. In the Examples will be found exercises on the formation of rules, *e. g.*, what easy method could be adopted for writing down the value of so many articles at £1 13s. 4d. each? The answer would be:

$$£1\ 13s.\ 4d. = £1\tfrac{2}{3} = £\tfrac{5}{3} = £\tfrac{10}{6}.$$

We must therefore imagine a cypher added to the number of articles, and divide by 6, which will give us the answer in pounds.

e. g., 477 articles at £1 13s. 4d. each = $£\frac{4770}{6}$ = £795.

124. By knowing the aliquot parts of the pound, shilling, &c., when we have found the units of pounds, if there are no shillings to add to the dividend, we can write down the fraction in shillings and pence from the sum for which we are dividing; *e. g.*, if we are dividing the value at £1 by 12 to find the value at 1s. 8d., and there is a remainder of 7, we need not change the 7 into 140 shillings, &c., but we can at once multiply the 1s. 8d. by 7, and write down the result as the quotient.

125. EXAMPLES XII.

Give rules or formulæ for writing down the value of so many articles at the following ten prices:—

1.	4s. 2d. each.	6.	6½d. each.
2.	£31 5s. ,,	7.	1s. 3½d. ,,
3.	19s. 2d. ,,	8.	3s. 9d. ,,
4.	2s. 1½d. ,,	9.	2s. 7½d. ,,
5.	4s. 7d. ,,	10.	3s. 1½d. ,,

Find the value of the following numbers of articles at the respective prices set against them:—

11.	512 at	4s. 2d.		31.	4,762 at	3s. 1½d.	
12.	317 ,,	£31 5s. 0d.		32.	5,918 ,,	6s. 3d.	
13.	211 ,,	19s. 2d.		33.	3,725 ,,	12s. 6d.	
14.	375 ,,	2s. 1½d.		34.	4,389 ,,	£1 5s. 0d.	
15.	476 ,,	4s. 7d.		35.	27,321 ,,	£16 13s. 4d.	
16.	327 ,,	6¼d.		36.	35,627 ,,	£83 6s. 8d.	
17.	259 ,,	1s. 3½d.		37.	472 ,,	£31 5s. 0d.	
18.	327 ,,	3s. 9d.		38.	3,724 ,,	£1 3s. 4d.	
19.	411 ,,	2s. 7½d.		39.	4,761 ,,	19s. 7d.	
20.	327 ,,	3s. 1½d.		40.	37,285 ,,	6¼d.	
21.	375 ,,	6s. 8d.		41.	32,549 ,,	1s. 3½d.	
22.	293 ,,	6s. 8d.		42.	37,216 ,,	2s. 1½d.	
23.	4,520 ,,	3s. 4d.		43.	3,721 ,,	£1 2s. 1½d.	
24.	3,072 ,,	10d.		44.	3,579 ,,	2s. 7½d.	
25.	57,681 ,,	1¼d.		45.	5,732 ,,	4s. 7d.	
26.	4,072 ,,	2s. 1d.		46.	36,921 ,,	5s. 0½d.	
27.	85,372 ,,	4s. 2d.		47.	8,732 ,,	5s. 7½d.	
28.	75,396 ,,	8s. 4d.		48.	38,271 ,,	9s. 6d.	
29.	37,216 ,,	16s. 8d.		49.	38,271 ,,	19s. 6d.	
30.	476 ,,	£1 13s. 4d.		50.	5,732 ,,	12s. 5½d.	

CHAPTER XIII.

SIMPLE INTEREST.

126. The usual rates at which the use of money is charged range from 2½ to 12½ per cent.

127. Since the sum paid for £1 at any rate, say 5 per cent., is $£\frac{5}{100}$, all we have to do is to multiply the sum of money given by this fraction $\frac{5}{100}$ or $\frac{\text{rate}}{100}$ and this fraction we must reduce, if possible, to one whose numerator and denominator are respectively 1 and a whole number; *e. g.*, to find the interest on £570 3s. 4d. at 2½ per cent. I must first mentally reduce $\frac{2\frac{1}{2}}{100}$ to $\frac{5}{200}$ or $\frac{1}{40}$, and write down $\frac{1}{40}$ of £570 3s. 4d. Since 3s. 4d. = 40d. and $\frac{1}{40}$ of 40d. = 1d., all I have to do in this case is to divide £57 by 4 and add on 1d., which gives me £14 5s. 1d.

128. If the numbers 20 or 12, or their multiples appear in the fraction we can often change multiplication, &c., into simple reductions. *e.g.*, 5 per cent. is $\frac{5}{100}$ or $\frac{1}{20}$. £396 5s. × $\frac{1}{20}$ = $£\frac{396\frac{1}{4}}{20}$ or $\frac{396\frac{1}{4} \times 20}{20}$ s. or 396¼ shillings; hence for 5 per cent. the interest is as many shillings and fractions of a shilling as there are pounds and fractions of a pound; hence £396 5s. at 5 per cent. is £19 16s. 3d.

129. We could have worked the example in 127 somewhat in this way: £570½ at 2½ per cent. is $\frac{2\frac{1}{2}}{100}$ or $\frac{5}{200}$ or $\frac{1}{40}$ of $£570\frac{1}{8} = \frac{570\frac{1}{8} \times 40 \text{ sixpences}}{20 \times 2}$ = 570⅛ sixpences = £14 5s. 1d.

130. Hitherto we have not taken the time into consideration. If the interest is for, say, three periods of time when the interest is, say, 4 per cent. per period, it is the same as 12 per cent. for 1 period, and therefore found by multiplying the sum of money or principal, as it is called, by $\frac{12}{100}$.

131. If we are asked for the interest for so many months, say 7, at 5 per cent. per ann., we must multiply the principal by $\frac{7}{12} \times \frac{5}{100}$. Suppose our principal is £250, our interest is $£\frac{250 \times 7 \times 5}{12 \times 100}$ or $£\frac{250 \times 7}{12 \times 20}$ or 250 × 7 pence; hence to find the interest at 5 per

cent. for so many months we can multiply the number of pounds in the principal by the number of months, and call the result pence—in this case 250 × 7 pence, or 250 × 6 pence + 250 pence, or 125 shillings + £1 0s. 10d., or £6 5s. + £1 0s. 10d., or £7 5s. 10d.

132. We might also find the interest in the question in 131, thus:

The interest on £250 for 1 year at 5 per cent. is £12 10s. and $\frac{7}{12}$ of this is £7 and $\frac{10 \times 7 \times 12}{12}$ pence, or 5s. 10d., hence, as before, our result is £7 5s. 10d.

133. If the interest is 6 per cent., and the period so many months, since 6 is contained in 12 twice, we have quite a simple fraction to multiply by: *e.g.*, what is the interest on £350 for 8 months at 6 per cent.? Our fraction is $\frac{8 \times 6}{12 \times 100}$ and our result $\frac{350 \times 8 \times 6}{12 \times 100}$ or £$\frac{350 \times 8}{2 \times 100}$ or £$\frac{350 \times \frac{8}{2}}{100}$ or £$\frac{7}{2} \times 4$ or £14.

which the student can reduce to a rule thus: Multiply the principal by half the number of months, and divide the result by 100, and this of course may reduce to a fraction whose numerator is 1.

134. If we have to find the interest for a certain number of days, say 23, at, say 4 per cent., we must multiply the principal by $\frac{23}{365} \times \frac{4}{100}$; we can reduce the fraction $\frac{23}{365}$ with advantage to $\frac{46}{7300}$, but unless the principal is a multiple of 73 such questions are very hard to perform mentally. Here is one: What is the interest on £1,460 for 29 days at 5 per cent.? Since 730 is contained in £1,460 twice, and 29 × 2 = 58, our question becomes the same as if we had been asked for the interest on £116 for 1 year at 5 per cent., which we can write down at once as 116s. or £5 16s.

135. If the rate is 6 per cent. we can easily get a close approximation to the interest, by calculating for 360 days to the year instead of 365; thus: What is the interest on £476 for 36 days at 6 per cent.? Our modified calculation is this:

$$£\frac{476 \times 36 \times 6}{360 \times 100} \text{ or } £\frac{476 \times \frac{36}{6}}{1000}$$

hence we get this rule: Multiply the principal by one sixth the number of days, and divide by 1,000.

136. Our answer in 135 is too large by $\frac{1}{73 \times 360}$ of £$\frac{476 \times 6 \times 36}{100}$ or £$\frac{\text{P.} \times \text{No. of days}}{100 \times 73 \times 60}$ or $\frac{\text{P.} \times \text{No. of days} \times 20 \times 12}{100 \times 73 \times 60}$ pence, or $\frac{\text{P.} \times 4. \text{ No. of days}}{7{,}300}$ pence; if therefore we subtract this small number of pence from the result in 135 we get the answer correct to 1 penny, ignoring the fraction or not, according as it is less or greater than $\frac{1}{2}$.

137. Find correct to a penny the interest on £750 for 27 days at 6 per cent. The mental operation is this: £$\frac{750 \times 4\frac{1}{2}}{1{,}000} - \frac{750 \times 108}{7{,}300}$ pence = £3 7s. 6d. − 11 pence + a small fraction of a penny = £3 6s. 6d. + a large fraction of a penny. The actual answer by slate arithmetic is £3 6s. $6\frac{88}{73}$d. We should therefore write down the answer as £3 6s. 7d.

138. I notice that very few, if any, of the examples given in the Government papers involve time as well as rate, and that the rate divided by 100 is always capable of being reduced to a fraction whose numerator and denominator are 1 and an integer respectively, unless the principal is a sum of money which is very easily divisible by 100; *e.g.*, in 1872 the male candidates were asked to find the interest on £3 400 16s. 8d. at 7 per cent.. Now £3,400 16s. 8d. divided by 100 gives us without the least difficulty £34 0s. 2d., and this multiplied by 7 gives the answer, £238 1s. 2d.

139. A student would do well to know the values in £ s. d. of the multiples of 100 pence—thus: 100 pence = 8s. 4d.; 200 pence = 16s. 8d.; 300 pence = £1 5s.; 400 pence = £1 13s. 4d.; ... 1000 pence = £4 3s. 4d.

140. EXAMPLES XIII.

Write down the simple interest on the following 20 sums at the rate placed by their sides:—

1.	£472 12s. 6d.	at 5 p.c.	11.	£4,301 13s. 4d.	at 7 p.c.
2.	£170 11s. 8d.	at 2½ ,,	12.	£7,504 3s. 4d.	at 3 ,,
3.	£250 18s. 4d.	at 2½ ,,	13.	£4,700 8s. 4d.	at 7 ,,
4.	£132 9s. 2d.	at 2½ ,,	14.	£742 17s.	at 6¼ ,,
5.	£863 15s. 4d.	at 6¼ ,,	15.	£124 2s.	at 3½ ,,
6.	£700 19s. 3½d.	at 4 ,,	16.	£133 19s. 4d.	at 3½ ,,
7.	£929 11s. 1½d.	at 4 ,,	17.	£861 17s. 6d.	at 7½ ,,
8.	£493 18s. 3d.	at 16⅔ ,,	18.	£616 5s.	at 6½ ,,
9.	£437 3s. 2½d.	at 4 ,,	19.	£729	at 33⅓ ,,
10.	£313 11s. 8d.	at 6½ ,,	20.	£426 12s.	at 16⅔ ,,

Write down the simple interest on the following 20 sums at the rates and for the times placed at their sides:—

21. £317 5s. at 2½ per cent. for 2 years.
22. £4,005 ot 3 per cent. for 2 years.
23. £400 at 5 per cent. for ⅝ years.
24. £2,504 3s. 4d. at 3½ per cent. for 2 years.
25. £250 1s. 8d. at 2½ per cent. for 2 years.
26. £3,703 15s. at 3 per cent. for 1½ years.
27. £1,803 6s. 8d. at 4 per cent. for 1½ years.
28. £780 4s. 8½d. at 2 per cent. for 2 years.
29. £1,260 7s. 6d. at 5 per cent. for ¼ year.
30. £143 15s. 10d. at 5 per cent. for ¼ year.
31. £360,198 19s. 6d. at 3½ per cent. for 1⅛ years.
32. £1,284 8s. 1½d. at 10 per cent. for 4 months.
33. £6,785 17s. 4d. at 2½ per cent. for 1 year 3 months.
34. £4,183 10s. at 3½ per cent. for 5 years.
35. £51 18s. 6½d. at 3 per cent. for 1 year 4 months.
36. £164 3s. 9d. at 2½ per cent. for 1 year 3 months.
37. £15,075 at 6⅔ per cent. for 2 years.
38. £346 11s. at 2½ per cent. for 2 years 6 months.
39. £44,609 3s. 9d. at 2½ per cent. for 1 year 3 months.
40. £369 at 6⅔ per cent. for 1 year 8 months.

Write down the interest on the following eight sums to the nearest penny at 6 per cent. for the times placed against the different sums:

41.	£2,400 for 4 months.	45.	£217 3s. 4d. for 48 days.
42.	£3,650 for 7 ,,	46.	£730 6s. 1d. for 17 ,,
43.	£4,302 for 42 days.	47.	£1,000 for 28 ,,
44.	£385 5s. for 6 months.	48.	£150 for 11 months.

49. If in calculating for 4 per cent. you considered there were 364 days in a year instead of 365 what would be the amount of the error on each pound for each day?

50. If in calculating for 3 per cent. you considered there were 363 days instead of 365 what would be the least number of pounds which would make the error 1s.—the number of days being 100.

CHAPTER XIV.

INTEREST, ETC., CONTINUED.

141. In the last chapter we only found the interest when the rate per cent. was given, nor did we consider any converse questions which arise from those on interest.

142. The amount of a sum of money at the end of a time, interest being charged at so much per cent. per period, is the technical name given for the sum of money increased by the interest thereon, *e.g.*, if the rate per cent. be 5, and the period one year, the amount of £100 at the end of 1 year is £105; at the end of 2 years, £110; at the end of 6 years, £130; and at the end of 20 years, £200, or double what it was at the commencement.

143. In finding the amount we reduce the fraction $\frac{\text{rate}}{100}$ to one whose numerator and denominator are respectively 1 and a whole number, and add it to 1, and multiply the principal by the equivalent improper fraction, thus: what does £250 amount to in 2 years at 5 per cent. The interest for 2 years at 5 per cent. is the same as for 1 year at 10 per cent. and $\frac{10}{100} = \frac{1}{10}$; we therefore have to multiply £250 by $\frac{11}{10}$, which gives us £275.

144. Sometimes we have the amount given to us from which we are asked to find the principal. By the well-known principles of reciprocity if £275 is obtained from £250 by multiplying it by $\frac{11}{10}$, we shall obtain £250 from £275 by multiplying it by $\frac{10}{11}$.

145. To work an example taken from a Government Paper: What is the principal on which the amount of 20 per cent. is £727 4s.; $1\frac{20}{100} = 1\frac{1}{5} = \frac{6}{5}$, and $\frac{5}{6}$ at £727 4s. is £606?

146. An income tax of 5d. in the £ reduces every pound or 240d. to 235d., the net income therefore is $\frac{235}{240}$ or $\frac{47}{48}$ of the nominal income, and the nominal income is $\frac{48}{47}$ of the net income; *e.g.*, on what income, after paying a tax of 5d. in the £ is £94 3s. 11d.? The student will see at once that £94 3s. 11d. ÷ 47 is £2 0s. 1d., which multiplied by 48 gives us £96 4s. as our gross income.

147. If the interest and the rate per cent. is given, we can find the principal by the principle of reciprocity, thus:

$$\text{Since interest} = \text{principal} \times \frac{\text{rate}}{100}$$

$$\text{principal} = \text{interest} \times \frac{100}{\text{rate}}$$

If the rate is 5 per cent. we have $\frac{100}{5}$ or 20. When multiplied by 20, we must remember that every shilling and fraction of a shilling becomes a pound, and the same fraction of a pound, which we can easily add to the pounds of the interest multiplied by 20; *e.g.*, on what sum is the interest £357 14s. 6d.? 14s. 6d. × 20 gives us £14 10s., hence the sum is £7,154 10s.

148. In what time will the interest on £119 12s. at $6\frac{1}{4}$ per cent. be £14 19s.? Since $\frac{6\frac{1}{4}}{100} = \frac{25}{400} = \frac{1}{16}$ and $\frac{1}{16}$ of £119 12s. = £7 9s. 6d., and £14 19s. contains £7 9s. 6d. twice, our answer is two years.

149. In what time would a sum of money double itself at $12\frac{1}{2}$ per cent.? the interest each year is $\frac{12\frac{1}{2}}{100}$ of the principal, and if $\frac{12\frac{1}{2}}{100}$ P × x = P.

$$x = \frac{100}{12\frac{1}{2}} \text{ or } \frac{100 \times 2}{25} \text{ or } 8.$$

150. If asked in how many years a certain sum would *amount* to another sum at any rate per cent. we must subtract the principal from the amount to find the interest, and then find the time as in 148.

151. EXAMPLES XIV.

On what sums are the interests given in the ten following questions, interest being calculated at the rate per cent. placed against each?

1.	£9	18s.	5d.	at	4½	p.c.	6. £27	11s.	9½d. at 2 p.c.
2.	£89	14s.	10d.	,,	10	,,	7. £8	7s.	6½d. ,, 1 ,,
3.	£18	15s.	7½d.	,,	2½	,,	8. £43	7s.	2d. ,, 12½ ,,
4.	£23	14s.	5d.	,,	1¼	,,	9. £24	12s.	5d. ,, 6¼ ,,
5.	£9	5s.	9½d.	,,	4	,,	10. £18	15s.	10½d. ,, 3½ ,,

What is the net income in the following six cases, if the income tax be the number of pence placed against each gross income?

11.	£720	6s.	8d.	I.T.	6d.	14.	£720	17s.	6d. I.T. 8d.
12.	£960	8s.	0d.	,,	5d.	15.	£960	6s.	8d. ,, 9d.
13.	£3,600	10s.	0d.	,,	4d.	16.	£480	8s.	0d. ,, 10d.

What is the gross income in the following six cases, if the income tax be the number of pence placed against each net income?

17.	£177	9s.	10d.	I.T.	4d.	20.	£319	16s.	11d. I.T. 8d.
18.	£235	19s.	7d.	,,	5d.	21.	£924	12s.	10d. ,, 9d.
19.	£273	6s.	6d.	,,	6d.	22.	£368	13s.	5d. ,, 10d.

What sums of money in the six following cases will amount in one year to the sum given, interest being calculated at the rate placed against each sum?

23.	£262	10s.	0d.	at	5 p.c.	26.	£3,411	7s.	6d. at 12½ p.c.
24.	£260	2s.	2d.	,,	4 ,,	27.	£243	11s.	3d. ,, 8 ,,
25.	£527	5s.	2d.	,,	3½ ,,	28.	£3,476	8s.	3d. ,, 10 ,,

At what rate per cent. is interest calculated in the six following examples if the different sums produce as interest the sums placed against them?

29.	£892	10s.	0d.	giving	Interest of	£17	17s.	0d.	
30.	£365	7s.	6d.	,,	,,	£18	5s.	4½d.	
31.	£700	19s.	3½d.	,,	,,	£28	0s.	9½d.	
32.	£929	11s.	1½d.	,,	,,	£37	3s.	7½d.	
33.	£897	8s.	4d.	,,	,,	£89	14s.	10d.	
34.	£6,618	1s.	8d.	,,	,,	£413	12s.	7½d.	

What is the *amount* of the six following sums at the rate per cent., and for the time placed against them?

35.	£750	2s.	1d.	at	4	per cent.	for	1	year.		
36.	£626	2s.	11d.	,,	6	,,	,,	2	years.		
37.	£2,000	8s.	4d.	,,	3	,,	,,	3	,,		
38.	£3,689	18s.	4d.	,,	2½	,,	,,	4	,,		
39.	£775	8s.	4d.	,,	4	,,	,,	2	,,	3	months.
40.	£1,025	4s.	2d.	,,	5½	,,	,,	2	,,	3	,,

41. What is the difference between the interest on £1,000 16s. 8d. calculated at 5 per cent. and at 6 per cent.?

42. What is the income tax payable on £960 4s. at 5d. in the pound?

43. On what income does the income tax of 7d. in the pound amount to £21 5s. 10d.?

44. The difference in a man's income is £3 0s. 1d. when the income tax is varied from 4d. to 6d. in the pound; find the income?

45. By how many pence in the pound is the income tax altered if on an income of £481 the net income is reduced by £2 os. 1d. ?

46. What is the difference on £481 between paying 5d. in the pound and 2½ per cent. ?

47. What is the difference on £405 16s. 8d. between paying 6d. in the pound and 3 per cent. ?

48. Find at 4 per cent. the interest on the amount of £100 at the end of one year at 5 per cent. ?

49. At what rate per cent. will the income tax on £961 be £4 os. 1d. more than what it is at 5d. in the pound ?

50. By what fraction must I multiply the amount of a given principal at the end of 2½ years at 3 per cent. so as to obtain therefrom the principal ?

(To be continued.)

Publications Received.

Baillière, Tindall, and Cox.

The Journal of Microscopy and Natural Science. July.
The Matriculation Directory. No. 2. June.

Bell, G., and Sons—

German Examination Papers in Miscellaneous Grammar and Idioms. By R. J. Morich.

Chapman and Hall—

Science and Art, No. 4.

Crosby Lockwood and Co.—

De Fivas' Elementary French Grammar and Reader.
The Civil Service History of England. By F. A. White, B.A., and H. A. Dobson.

Curwen, J., and Son—

Cantata: The Coming of the Flowers. By George F. Root.

Fisher Unwin, T.—

The Story of the Nations: Hungary. By Prof. Vambéry.

Heywood, John—

The Swiss Family Robinson.

Longmans and Co.—

Open Competition Handbooks. By W. J. Chetwode Crawley, LL.D.: Preliminary Arithmetic.
Wonderful Nests. By the Rev. J. G. Wood, M.A.
Sea Monsters and Sea Birds. By Dr. G. Hartwig.
Handbooks of English. 1 to 6.
Longmans' New Geographical Reader. Standard I.

Roberts, J., and Sons—

The Formation of English Writers. By C. C. Longridge.

Swan Sonnenschein and Co.—

Education in the Home, the Kindergarten, and the Primary School. By Elizabeth P. Peabody.
Exercises in English Word-Formation and Derivation. By F. Ritchie, M.A.
Greek Verbs in a Fortnight. By John Carey, B.A.

Ward, Lock, and Co.—

Self-Instruction in German. By H. W. Dulcken, PhD.

Publications Reviewed.

On the Teaching of Modern Languages in Theory and Practice. By C. Colbeck, M.A. Cambridge University Press.

This little book consists of two lectures delivered in the University of Cambridge at the request of the Teachers' Training Syndicate. The author believes that at last there is a disposition to regard seriously the pretensions of modern languages to a larger place in education; and he has striven to promote their claim, and especially their claim as literature, to a fair hearing.

We cannot praise these lectures too highly. They seem to us to be the perfection of common sense as applied to the teaching of modern languages. It is impossible in the short space at our disposal to do justice to the author by quoting a sentence here and there. The difficulty would be in selecting the paragraphs which were not to be quoted. In the first lecture the author discusses the reasons for teaching modern languages and the systems by which this has hitherto been done. He also attempts to answer the question—should the teaching be mainly by English or native teachers.

The author modestly tells us in the preface that he does not think any one but a teacher will read the second lecture to the end. That may be so, but we will venture to say that no teacher who begins it will fail to read the second lecture to the end, whether he has tried to teach modern languages or not, and if he has he would not be content with reading it once or even twice.

It is not often that a really competent man will tell his fellow workers what he conceives to have been the secret of his success. Mr. Colbeck has done this and he has done it admirably. He deserves, and we are sure he will receive, the sincerest thanks of hundreds of teachers. We advise every one who takes an interest in the teaching of French or German to get this book without delay. It is altogether the freshest, most stimulating and suggestive attempt to advance the study of modern languages that has been published for many a day.

Questions in Psychology, Metaphysics and Ethics. By J. Ryland, M.A. London: Swan Sonnenschein and Co.

The author tells us that the object of this book is to provide students, and especially those who have the misfortune to be candidates for examination, with a collection of questions which may serve as a guide for study, a test of proficiency, and a stimulus to inquiry. The questions are taken from papers set during the last dozen years at examinations at the Universities of Cambridge, Oxford, London, Edinburgh, and Dublin, and also at some of the examinations of University College and the Civil Service. There is an excellent table of contents, by means of which the student can at once turn to a set of questions upon nearly any of the important points in Psychology, Metaphysics and Ethics. Mr. Rylands has produced a very useful book. It is often exceedingly convenient to be able to find a few really good questions upon a given subject. Whether a student is studying for an examination or not, he will be able to turn this collection of about 1,400 questions to good account. The book is clearly printed upon good paper and neatly and strongly bound.

A Short German Accidence and Minor Syntax. By H. W. Eve, M.A. London: D. Nutt.

The German grammar of Mr. Eve is well known and much liked. The volume before us consists of only 100 pages. It contains the accidence and minor syntax from the larger grammar. The author hopes it will be found sufficient for a boy in the fifth form. It is uniform with the Wellington College French Accidence. To those who are familiar with the larger work the present volume will stand in no need of an introduction. The types are

clearly printed, and the exceptions seem to be pretty fully given. We hope they are not meant to be learnt, though the rhymes into which they are often thrown seem to imply that they are to be committed to memory. The teacher can however decide for himself whether he will do more than use them for reference tables, for which they are of course admirably adapted.

The syntax which only occupies about one-tenth of the book is a plain statement of the most important points which a boy should have at his finger's ends.

This is not quite the book for a private student; but for a class with a good teacher it is admirably adapted, not less for what it contains than for what it omits.

Stanford's Intermediate School Physical and Political Atlas. London: Edward Stanford.

This Atlas contains new and striking features. The general maps of continents, both physical and political, are admirable specimens of the art of chartography. The selection of matter is most judicious, and at the same time the style and artistic merit are excellent.

We are most pleased with the smaller maps. These record all the geographical facts a youth will ever need to learn, and present the information in so simple a form that he will be stimulated to imitate them.

Atlas of the British Empire. New and Enlarged Jubilee Edition. By John Bartholomew, F.R.G.S., 3s. 6d. London: Geo. Philip and Son.

This is a most comprehensive Atlas. It satisfies every requirement of the students. The amount of matter on each map, and the number of maps, make it a complete handbook of reference for the geographical student.

The Colonial and Indian Atlas of the British Empire. London and Edinburgh: W. and A. K. Johnston.

For minute detail, for complete accuracy, and for clearness and style, we know of no Atlas of British Geography that can compare with the Colonial and Indian Atlas. No student of the British Possessions should be without it.

Geography of the British Isles, with definition of Mathematical, Physical, and Political Geography. London: Geo. Philip and Son.

The matter in this little text book is carefully selected, well arranged, and thoroughly accurate.

Home Lesson Book. Fourth Historical Reader. London: Blackie & Son.

This is a full historical summary suitable for the class subject 'History' for the higher standards, and embraces the period from the accession of the Stuarts to the present time. It follows in succession the paragraphs of the Fourth Historical Reader, the subject matter of the paragraphs and the dates being printed in bolder type. The difficult words of the lessons are all explained, and the connection between Geography and History is shown by a series of geographical notes appended to each lesson. The book will well answer all the purpose for which it was compiled.

Handbook of the New Code, 1887. By John F. Moss. London: Cassell & Co.

We were constrained to speak in terms of sincere approval of Mr. Moss's edition of the code on its first appearance, and this year's edition confirms our high opinion of the care with which it has been compiled. All the difficult points which experience of the working of the code have shown are here elucidated, letters and other documents of the Education Department being inserted where necessary. The recent circulars on Drawing are appended. It is no fault of the compiler that the instructions to Her Majesty's inspectors which it contains are now out of date. Publishers and editors alike have serious reason to complain, along with managers and teachers, of the remissness of the Department in issuing these instructions three months after the code which they are supposed to explain has been in the hands of those whose work it regulates. However, as most inspectors apparently interpret the code in their own way, no serious difficulty will arise, and the defect will, no doubt, be remedied by the insertion of the changes at the earliest opportunity.

Shipwrecks and Tales of the Sea. London and Edinburgh: W. & R. Chambers.

A collection of narratives of many famous shipwrecks with other tales of the sea, has been published as an attractive prize, or book suitable for school or other libraries, by Messrs. Chambers under the above title. The sea, notwithstanding its dangers, still maintains its fascinating influence over English people, and the tales of fortitude under difficulties, and in times of extreme peril, as well as the records of adherence to duty, contained in this volume, cannot but be of service, not only in affording pleasure and instruction, but also in strengthening or developing the same characteristics in all who peruse them.

What and How: A Guide to Successful Oral Teaching. By Joseph Hassell, A.K.C. London: Blackie and Son.

Under the present regime, where intelligence is so much needed in the management of schools, this little volume will be of great service. It is a short, clear, and concise guide to successful oral instruction. The causes of failure in oral teaching are pointed out, and clear pithy rules are laid down for the formation of questions and the use of illustrations. A number of pattern 'Sketches of Lessons' are added which are models of simplicity, style, and direction of purpose in the work in hand. The examples of bad lessons and faulty questioning are taken from actual specimens collected during the time the writer was assistant master of method in a training college, and teachers of experience will find here ready to hand corrections of the errors which they daily meet with in their junior teachers. To pupil teachers, students in training, and junior teachers generally the pithiness and directness of the teaching of this little volume will be invaluable.

Poynter's South Kensington Drawing Book. London: Blackie and Son.

We have already had occasion to speak favourably of this work. The new edition before us is issued on fine cartridge paper, and contains detailed instructions and space for copying every example. The portions of the work now under review include six books of Freehand First Grade, two of Freehand Elementary Design, six of Freehand First Grade—Plants, and four Freehand Second Grade. Of the drawing, the paper, and the course generally we can only speak in terms of the highest praise. Any student who has thoroughly mastered the series will have no mean capacity for drawing, but will be well furnished with the elements necessary for the pursuit of any class of art for which he may have a taste. The series deserves our highest commendation.

Story of the Nations: Hungary. By Arminius Vambéry. London: T. Fisher Unwin.

The author of this addition to the above story is a Professor at the University of Buda-Pesth. He tells us in his preface that the work is new to him, as hitherto his literary labours have been devoted to the geography, &c., of Central Asia. It is only just to say, in passing, that Professor Vambéry's English is as fluent and free from any signs of its being anything but his native language as can be.

With regard to the work itself the professor is justly proud of his country, which has had a continuous kingdom for nearly 1,000 years, and writes as a patriot.

The pretty story of Lehel and his reputed ivory bugle-horn reads as a relief amidst the less interesting pages, in which the events of a considerable time have often to be compressed into a few lines.

The history is brought down to the present time.

As this is the first History of Hungary that has ever been written in English, we need not shrink from saying that we believe that it is a subject very little known to Englishmen, and that therefore this volume will be specially welcome to them, being thoroughly worthy of their confidence.

The Cornhill Magazine. July. London: Smith, Elder and Co.

This excellent magazine still maintains its high reputation. It is full of 'good things' and well worth the modest sixpence at which it is issued.

The Gospel of St. Luke. Part II., with Notes, &c., by T. M. Lindsay, D.D. Blackie and Son.

These 'Commentaries for Bible-classes and senior sabbath schools' are worthy of the great firm that issues them. Unlike many of these cheaper manuals on different parts of the Bible, they contain the English version of the portion included, which is printed in slightly smaller type above the notes.

We notice that Dr. Lindsay has an idea of teaching his readers a good deal more than what would be expected of them to reproduce in an examination (even of a diocesan inspector)—as he wishes to lead his young readers to study their Bible as a book by which they may regulate their conduct and test their faith.

The Story of the Nations: Ancient Egypt. By Prof. Geo. Rawlinson. London: T. Fisher Unwin.

Again Mr. Unwin has been particularly happy in the selection of his author for this fresh portion of his story. Few, if any, Englishmen know more than he of this interesting country, which has ever been, even as it is now, an important factor in the doings of the world.

We have at the beginning of the volume lively descriptions of the country, and sketches of the probable and possible origins of the people, with a learned and instructive disquisition on its religion.

In the chapter entitled 'The Dawn of History' we have some of the old myths of Egypt, which the reader will find intensely interesting.

What the strict believers in the verbal inspiration of the Sacred writings will say to the professor's account of that portion of the history of the Exodus which relates to the passage of the Red Sea by the Israelites, we hardly dare to contemplate; but they will have to thank him for drawing attention to the fact that the Pharoah of the Exodus mentioned in an inscription, still in existence, the loss of his eldest son. Biblical students would have been glad to have had this inscription—or such part of it as referred to what we learn from the books of Moses—given in its exact words.

With the exception of the story of the passage of the Red Sea, the professor seems to accept the old Biblical story, freely quoting therefrom.

In discussing the pyramids, and their origin and use, we notice the author entirely ignores the modern theories as to the mystic and prophetic meaning of the Great Pyramid, which will disappoint some people considerably.

The illustrations, which are numerous, are principally from ancient sculpture, but there are a few pictures of Egypt as it looks now.

We have the usual index, and altogether the work is as complete as the careful work of an able man can make it, and as such we heartily recommend it to all readers and students of ancient history.

New National Copy Books. National Society's Depository, Sanctuary, Westminster.

These books are printed on very good paper with a smooth surface, and the copies are generally bold, clear, and well written, the earlier numbers being supplied with suitable aids. The only fault we have to find with the writing is that it becomes too small in the higher numbers.

Syllabus of Elementary Geometrical Conics. London: Swan Sonnenschein and Co.

This syllabus is prepared by the committee appointed by the Association for the Improvement of Geometrical Teaching.

We confess we do not quite understand the object of this publication, nor has it any preface to tell us.

We can scarcely believe that it is printed and published for ordinary sale, because if so we cannot imagine to what class of readers or students it would be of the slightest use. We could, of course, understand its being printed as an advertisement for the purpose of giving teachers an idea of what properties of the curves are going to be treated of, and in what order; or we could understand the committee sending it out for the purpose of obtaining suggestions from others, as experienced in education as they, with regard to the proposed plan. Had, however, this last been their idea we should have expected some request or hint as to this wish on their part.

The volume as it stands consists of 32 pages, with three full-page figures capable of being undoubled so as to be visible for several pages. These pages, which in some cases only have some eight or nine lines of printing on them, are interleaved with writing paper, as if courting notes and remarks.

Since there is only one figure for each of the three conics, and all the propositions of each curve are referred to this single figure, the number of lines, &c., especially in that of the hyperbola, is excessive and somewhat confusing, though we gladly add that the figures are exquisitely printed, and angles which ought to be right angles are, though not perfect right angles, more nearly so than we often find in such treatises.

After the parabola there is a page of definitions and enunciations with respect to orthogonal projection, but no figure illustrates them; and after the hyperbola there are some six pages of definitions and enunciations with respect to the cylinder and cone, but here again we have no figures to illustrate them.

If it is not too late to make suggestions, we cannot help thinking that Mr. Besant's plan of collecting at the commencement of the volume those properties which are common to all conics might with great advantage be adopted, and we are surprised beyond measure to find no allusion whatever to the circle of curvature at a point, without a knowledge of which we should venture to think that no one is properly grounded in the science.

Geometrical conics have not, like the geometry of triangles, circles, &c., an expositor whose methods and sequences will probably be those accepted and taught by the higher class of teachers as long as the world lasts. We need not, therefore, examine with special jealousy any deviation from the order, &c., now in use, nor have we any fault to find with that proposed here.

Much as we admired the clearness of the figures we cannot help adding that if in the work itself, which we presume is coming out, the committee only intend to have a single figure of each conic they ought to try and get a perfect curve, so that the angles may really be what they pretend to be. It is very hard for a young student to realise and remember that an angle drawn on a plane surface containing some 75° or 80° is really a right angle, not that the angles in this manual are anything like as much out as that.

We can only say, in conclusion, we look for the work itself with interest, and shall give it our careful consideration.

The Child's Bible Expositor. By S. E. Scholes. Manchester: John Heywood.

The author tells us that the principal object of this pretty little volume of 190 pages is 'to awaken in the minds of the young a spirit of interest and inquiry as regards the Old Testament;' and though the New Testament is not altogether neglected, there are only some 40 pages bearing on it.

The information given is in the form of Question and Answer, of which we will give three specimens, in one of which the information given is certainly wrong, or at least different from what we find in the book which the volume is supposed to be expounding, in another of which the information is very doubtful, and in the third of which the information is undoubtedly correct:

Q. What did God create on the fifth day?

A. All cattle and creeping things.

Q. Why was the Tower of Babel built?

A. Because the descendants of Noah feared a second flood, and thought they might, by means of this high building, be kept from destruction.

(Gen. xi. 4.)

Q. Who told Ahab he had no need to be sorrowful, for he should yet have Naboth's vineyard?

A. His wicked wife Jezebel, who at once sent letters, in Ahab's name, to the nobles of Jezreel, commanding them to proclaim a fast, and then, accusing Naboth of blasphemy against God and the King, to stone him to death.

(1 Kings xxi. 8—11.)

Now of these three answers, the first and third require no comment; with regard to the second, many of us, no doubt, were either taught, or maybe imagined in our childhood that protection against a second flood, even after the direct promise of God on the subject, had something to do with the erection of the huge and lofty tower, but surely the very verse referred to in the answer shows that such was not the case.

There are three pages devoted to the Psalms, explaining terms like Selah, Sheminith, and Michtam, and a few expressions such as 'Dogs have compassed me,' 'A portion for foxes,' &c., but there is not one word about the fact of there being five Books of Psalms, or of the construction of the 119th, or of the Alphabetical Psalms. The two pages devoted to the Book of the Prophet Isaiah are certainly not used to teach the most important things that a student, however young, might be expected to know, that is, if he or she could be expected to know anything at all about such portions of the Bible.

The book, having passed into its seventh edition, has, we presume, been accepted by a goodly number of people, but our readers will see that they would have to be careful into whose hands it fell, as it might be used by some who knew no better themselves, to teach that which a student would have to unlearn.

Combination Exercise Book for Analysis and Parsing, with full notes and numerous illustrative examples. Price 1s. Relfe Brothers.

Another novelty, well conceived and carefully executed. This exercise book is designed to assist the scholar and at the same time save the time of the teacher. The notes on analysis and the forms for parsing are very complete, and as they are printed on the covers of the exercise book, they may be retained for use after the book is filled.

The Mundella Drawing Book. By George Oscar Blacker. Manchester and London: John Heywood.

This drawing book is published in nine numbers, and contains copies, instructions and space for drawing in each number. The examples are clearly and beautifully printed, and are admirably suited for the higher classes of schools, but are too advanced and difficult for ordinary classes in elementary schools.

Science and Art Department Examination Questions in Animal Physiology, 1869 to 1886. Arranged and classified by J. Levy. Hamilton, Adams and Co.

Those science teachers (and they are many) who have experienced difficulty in obtaining the questions of past years in Physiology will find the present little volume a great boon.

The questions of 17 years are arranged in groups according to the system to which they belong (circulatory, digestive, muscular, &c.); and the elementary questions are classified separately from the advanced.

We have no sympathy with those science teachers who do not conscientiously prepare their pupils, but, by a study of the questions of the few preceding years attempt by a process of elimination to discover the questions that must be given *this* year, and devote inordinate attention to these. Such a plan is generally followed by the failure it deserves. But a legitimate use of questions given in past years may be made, and especially so when a large number of questions are gone over. As Mr. Levy says, 'a very large proportion of the failures at the May Examinations is owing not so much to a want of knowledge as to inexperience in writing and in setting out that knowledge to the best advantage—a qualification only to be acquired by regular practice before the final examination.'

We heartily commend this volume to the notice of science teachers, as containing so far as we know the only classified collection of questions in Animal Physiology for a long series of years.

Easy Latin Passages for Unseen Translation. By A. M. M. Stedman. London: Geo. Bell and Sons.

We have more than once had the pleasure of drawing our readers' attention to the works and compilations of Mr. Stedman.

This little work consists of passages, carefully selected according to difficulty and length (from two lines to half a page), which the student is expected to translate without the aid of a Dictionary or Grammar. Such an exercise forms an item of every well arranged curriculum, and masters will find it very useful for the younger classes. Though prose passages are generally given, there are a considerable number of extracts from the Poets. We cannot help, however, thinking it a great pity that the author did not add the source of each extract. It is a great satisfaction to a little boy to know that he has been able to make something of an author whose works he fancies are far beyond his power.

Crustacea and Spiders. By F. A. A. Skuse. London: Swan Sonnenschein and Co.

This is a most valuable little work, treating, as it does, of creatures which are seldom studied, owing principally to the want of trustworthy information regarding them. Mr. Skuse begins from the very beginning, assuming little or no knowledge upon the part of his reader, and leading him gradually on from the mere rudiments of the science to the more advanced details, which to a beginner would be both incomprehensible and useless. A short account is given of crustaceans and spiders as groups, followed by suggestions upon the arts of capturing and preserving them. Then come chapters upon development, habits, and habitats, succeeded by a brief outline of the system of classification at present in vogue. And, finally, there are a few words upon the uses and influence of such animals as those described.

With one notable exception, too, the illustrations are thoroughly good. The representation of the water spider and its nest upon p. 108, however, is altogether misleading and incorrect. As every one who possesses any personal knowledge of the spider must be aware, the nest is always built *among* water weeds, not moored down in the open

water to a couple of leaves after the manner of a captive balloon. And the proportions of the nest, as illustrated, are greatly exaggerated, while the spider invariably rests with its head downwards. This, however, is not a matter of any great importance, and can scarcely be held to detract from the value of the book.

We are glad to notice, in the chapter upon *Collecting*, a strong protest against the mania for accumulating specimens which so often poses under the name of science, and leads to great and constant destruction of life for no real and useful purpose. 'If his only object,' says Mr. Skuse, with reference to the collector, 'is to merely accumulate a number of specimens to which he can attach Latin names—if his collection is to be regarded in the same way as a collection of coins or postage stamps—then let him satisfy his propensity for gathering together a variety of objects in the function of a stamp collector, for he lacks the spirit of a naturalist.' Would that such sentiments were more generally entertained and carried into practice.

Shakespeare's Play of A Midsummer Night's Dream. By F. C. Woodforde, B.A. Market Drayton: Bennion and Horne.

The selection of this play for the English of the Cambridge Locals has led to its publication in this form. It consists of a copy of the play with notes and glossary compiled from Halliwell's Shakespeare and Skeat's Etymological Dictionary. The notes are pretty full and the glossary large and complete. The type is small and the get-up falls far behind that of most school books of the present day.

The Holborn Series of Registers. Class Register of Attendance and Fees: No. 2. Educational Supply Association, Limited: 42a Holborn Viaduct, London, E.C.

This new register has spaces for sixty names, is without fly-leaf, and is very strongly bound in stiff boards on cloth-backed guards. The ruling is remarkably clear and distinct, and while they contain no useless or obsolete forms, the information provided for is all that can be desired. Pence columns are kept separate.

The publishers are to be congratulated on the production of such a good register, and may fairly lay claim to its being one of the strongest and most durable in the market.

Owing to pressure on our space we are compelled to hold over 'Hints to Pupil Teachers,' and particulars of the Prize Scheme, as well as a large number of Reviews.

The 'Practical Teacher' Sale and Exchange Column.

Post Free. Good as New.—Fowne's Inorganic Chemistry, edited by Watts (8s. 6d.), 4s. ; London University Calendar, 1885 (4s.), 2s. 6d. ; Stewart's Deductions, and Rider's Euclid, I.—IV. (3s. 6d.), 2s. ; The English Language, Grammar and History, by H. Lewis, Battersea College (3s.), 1s. 6d. ; Dodd's Guide to Matriculation (1s. 6d.), 1s. : Lee's Sound, Light, and Heat (2s. 6d.), 1s. 3d. ; Stewart's Guide to Certificate and Scholarship (2s. 6d.), 1s. ; Moffatt's Mental Arithmetic (1s. 6d.), 9d.—Mitchell, Waghen, S. Sutton, Hull.

EYRE AND SPOTTISWOODE,
THE QUEEN'S PRINTERS.

THREE MOST SUCCESSFUL BOOKS.

'THE QUEEN'S PRINTERS'
TEACHERS' BIBLE.'
The ORIGINAL,
FULLEST,
And BEST.
ELEVEN SIZES, from 3s.

'THE VARIORUM
TEACHER'S BIBLE.'
With Foot Notes
BY
CHEYNE, CLARKE, DRIVER, GOODWIN, and SANDAY.
From 7s. 6d.

'BARRY'S
TEACHER'S PRAYER-BOOK.'
By Dr. BARRY, Bishop of Sydney and Primate of Australia.
Prayer-Book and Notes interpaged throughout.
NEW EDITION, with Notes on the Psalter, by Bishop BARRY, and Glossary by MAYHEW.
Large type, 16 mo. ... ... from 6s. 0d.
Nonpareil type, 24 mo. ... ... ,, 3s. 6d.
School Edition (without Notes on Psalter and Glossary) ... ... ,, 2s. 6d.
The Bishop of CHICHESTER wrote:—'Canon Barry was of all others the most fitted and best qualified for such a task.'

The Sale of the above Books is so large, that Reprints are constantly being required.

Just Ready. An Edition upon Thin India Paper.

VARIORUM TEACHER'S BIBLE.

Nonpareil 8vo. ($7\frac{3}{4}$ × $5\frac{1}{2}$ × 1 *inches*) *1250 pages.*

THE VARIORUM NOTES serve as AN INDEX TO THE REVISERS' CHANGES in both Testaments, and for practical use combine a Revised with the Authorised Version.

'Gives, as nearly as possible, a correct idea of the meaning of the original text.'—*Saturday Review.*
'The footnotes "give the sense" of hundreds of passages more or less obscure, and save time and labour in referring to commentaries.'—*The Christian.*

Printed upon India paper of the finest texture, reducing the bulk of the volume, so that the 450 additional pages, comprising the 'Aids to Bible Students,' are included within the dimensions of a book no thicker than an ordinary Reference Bible.

The original 'VARIORIUM TEACHER'S BIBLE' is most highly prized by those who use it. Where the well-known 'Revised Version' makes such references as 'Some ancient authorities read,' these ancient authorities are named in the above work.

The notes appeal at once to the ordinary Bible reader, whose chief difficulties they endeavour to meet, and to the special or professional student, who will find, it is hoped, particularly in the Old Testament, a more careful selection of critical data and authorities than is elsewhere accessible.

Prices of the New India Paper Edition—from 13s. 6d. to 35s.

EYRE AND SPOTTISWOODE, GREAT NEW STREET, FETTER LANE, LONDON, E.C.
RETAIL OF ALL BOOKSELLERS.

GEORGE BELL & SONS' BOOKS.

December, 1887. **CERTIFICATE EXAM.**

CICERO.—DE SENECTUTE. English Notes by GEORGE LONG, M.A. 1s. 6d.

EURIPIDES.—HERCULES FURENS. English Notes by F. A. PALEY, M.A., LL.D., late Classical Examiner at London University. 1s. 6d.

XENOPHON.—ANABASIS. By the late J. F. MACMICHAEL, Head-Master of the Grammar School, Ripon. Revised Edition. Books II. and III. 1s. 6d. *(Passages from Book II. set.)*

VIRGIL.—ÆNEID. With English Notes. Abridged from Professor Conington's Edition by the late Dr. WAGNER, Books XI. and XII. 1s. 6d. *(Passages from XII. set.)*

January, 1888. **LONDON UNIV. MATRIC. EXAM.**

*HORACE.—THE ODES. English Notes by the late A. J. MACLEANE, M.A. Fcap. 8vo, cloth, 2s. 6d. *(Books III. and IV. set.)*

*EURIPIDES.—ANDROMAQUE. English Notes by F. A. PALEY, M.A., LL.D., late Classical Examiner at London University. 1s. 6d.

June, 1888.

*CÆSAR DE BELLO GALLICO. Books VI. and VII. Edited, with English Notes, by the late GEORGE LONG, M.A. 1s. 6d. *(Book VII. set.)*

*XENOPHON.—CYROPÆDIA. Books I. and II. Edited, with English Notes, by G. M. GORHAM, M.A., late Fellow of Trinity College, Cambridge. 1s. 6d. *(Book I. set.)*

* *Set* ALSO FOR CERTIFICATE EXAMINATION, December, 1888.

STEDMAN'S EXAMINATION SERIES.

Crown 8vo, 2s. 6d. each.

FRENCH PAPERS in miscellaneous Grammar and Idioms. By A. M. M STEDMAN, M.A., Wadham College, Oxon. 2nd edition, revised. In use at Eton, Harrow, Winchester, Cheltenham, Haileybury, etc.

GERMAN PAPERS in miscellaneous Grammar and Idioms. Compiled by R. J. MORICH, Manchester Grammar School.

LATIN PAPERS in miscellaneous Grammar and Idioms.

GREEK PAPERS. (In the press.)

QUESTIONS FOR EXAMINATION IN ENGLISH LITERATURE. Chiefly selected from College Papers set in Cambridge. With an Introduction on the study of English. By the Rev. Professor W. W. SKEAT, Litt.D., LL.D. Second Edition, revised, crown 8vo, 9s. 6d.

EUCLID. Books I.—VI., and part of Books XI. and XII., newly Translated from the Greek Text, with Supplementary Propositions, Chapters on Modern Geometry, and numerous Exercises. By HORACE DEIGHTON, M.A., Head Master of Harrison College, Barbadoes, 4s. 6d. Books I. and II. 2s.

HYDROSTATICS, ELEMENTARY. W. H. BESANT, D.Sc. Twelfth Edition. Revised. 4s.

GEOMETRICAL OPTICS, an Elementary Treatise on. By W. S. ALDIS, M.A., Professor of Mathematics in the University College, Auckland, N.Z. Second Edition, Revised. 4s.

HEAT, an Elementary Treatise on. By WILLIAM GARNETT, M.A., D.C.L. Fourth Edition, Revised and Enlarged. 4s.

EXPERIMENTAL CHEMISTRY, founded on the work of Dr. J. Stöckhardt. By C. W. HEATON, F.I.C., F.C.S. New Revised Edition, Small Post 8vo, price 5s.

LONDON: GEORGE BELL AND SONS, YORK STREET, COVENT GARDEN.

ALL UNIVERSITY, PROFESSIONAL, HIGHER AND LOWER CIVIL SERVICE, ARMY, AND NAVY EXAMINATIONS:—Most rapid and successful preparation in the United Kingdom (proved by opinion of the Press, Testimonials, Recent Successes, &c.), by Messrs. TILDESLEY, GRASSIE, & Co., United Kingdom College, Norton Park, Edinburgh, assisted by a large staff of distinguished and highly successful Coaches and Specialists in all subjects. Complete list of coming Examinations and existing vacancies appear weekly in the *Higher 'Exam.' Coach and Civil Service Aspirant*; Weekly Journals, Reviews, and Coaches for above examinations. Indispensable to all Candidates for the above examinations. Price, 1d. each. Specimens of each, with Guide and Prospectus, *gratis*. Write at once (full address), Secretary, United Kingdom College, Norton Park, Edinburgh.

ALBION COPY BOOKS.
ALBION EXERCISE BOOKS.
ALBION SCHOOL CERTIFICATES.

SCHOOLS SUPPLIED BY

THOMAS LAURIE,
LONDON SCHOLASTIC TRADING CO.,
28, PATERNOSTER ROW, LONDON.

MURBY'S IMPERIAL COPY BOOKS.

'EXCELLENT.' *Rapidly finding their way into all good Schools.* 'UNEXCELLED.'

17 NUMBERS, PRICE 9d. EACH.

1. Half Text.	5½. Large and Small.	9. Small Hand.
2. Half Text.	6. Small Hand.	10. Small Hand.
3. Half Text.	6½. Double Small.	11. Small Hand.
3½. Large and ½ Text.	7. Small Hand.	12. Small Hand.
4. Half Text.	7½. Three Hands.	12½. Ladies' Hand.
5. Small Text.	8. Small Hand.	

London: THOMAS MURBY, 3, Ludgate Circus Buildings, E.C.

UNIVERSITY CORRESPONDENCE COLLEGE

(With which the Intermediate Correspondence Classes are connected).

TUTORS.—**M.A.**—Classics and A.S. **M.A.**—Philosophy. **B.Sc.**—(Hons.) **M.B.**—(Hons.), Sub-Examiner of Lond. Univ. **B.A.**—(Lond.) Double Hons.—French and German. **M.A.**—Camb. and Lond. Wrangler. **M.A.**—Wrangler, late Examiner. **LL.B.**—First in First-Class Hons.

SYSTEM.—*Weekly* Schemes of Study, Notes, Hints, Vocabs, and at week end a Test Paper, *compiled from previous Exam. Papers*, and solution of difficulties. Most successful.

Single subjects, or full preparation for **MATRIC.**, **INTER.**, and **FINAL B.A.**, at low fees, payable as arranged on joining.
Students for Intermediate Arts, 1888, are requested to join early.
Full particulars given on application to—

Prof. CLIVE, 54, Nelson Square, London, S.E.

BOOKS BY THE TUTORS.

INTERMEDIATE ARTS GUIDES (Price 1s). Contain Questions of last Exams., with full Solutions to Mathematics. No. 2 ready end of July.
'Full of that particular kind of information so needful to those about to undergo examination.'—*Teachers' Aid.*
'Really useful, nothing can be better for the student who intends to present himself next July.'—*School Guardian.*
'An excellent selection of Text Books.'—*Practical Teacher.*

MATRICULATION DIRECTORIES (Price 6d.). On same lines. No. 2 end of June.
'Come (like the Pickwick Pens) "as a boon and a blessing."'—*School.*

INTERMEDIATE MATHEMATICS (Price 2s.) contains Questions of last twelve Exams. and all Conics since 1840, Answers and Hints for Examination, Structure of Papers, &c.
'Of very special value to the student preparing for the Intermediate. Everything is "cut and dry" in the very best sense.'—*Educational News.*

MATRICULATION MATHEMATICS. Just Issued, Price 1s. 6d On same lines.
'An excellent daily companion for the candidate.'—*School Board Chronicle.*

B.A. MATHEMATICS (Pure). Price 2s. Contains Solutions to every Paper set at that Examination.
'The solutions are admirable, and cannot fail to be suggestive even to experienced mathematicians.'—*I. Teachers' Journal.*

Post free, for stated amounts, from Prof. CLIVE.

The Practical Teacher.

A MONTHLY EDUCATIONAL JOURNAL.

To Subscribers.—The *Practical Teacher* is published on the 25th of every month. Price 6d.; post free, 7½d.; sent post free, three months for 1s. 11d.; six months, 3s. 9d.; a year, 7s. 6d.

P. O. Orders should be made payable at Chief Office.

Subscribers not receiving their copies regularly are respectfully requested to write to the Publisher.

It would save time and expense if subscribers when remitting would state exactly what numbers they wish to be sent.

To Correspondents.—All literary communications should be addressed, THE EDITOR OF THE PRACTICAL TEACHER, Pilgrim Street, Ludgate Hill, London, E.C.

Accepted contributions are paid for within twenty-one days of publication.

The Editor cannot return rejected MSS.; authors should therefore retain copies.

TO ADVERTISERS.

Approved, *prepaid*, advertisements will be inserted in the *Practical Teacher* at the following rates:—

	s.	d.
20 words or under...	3	0
Above 20 words and under 32 words	4	0
32 words and under 64 words	5	0
For every additional 10 words...	1	0

For an ordinary page	4 guineas.
For page facing matter	5 guineas.
For 2nd page of wrapper	5 guineas.
For 3rd page of wrapper	5 guineas.
For the page opposite the 3rd page of wrapper...	5 guineas.
For the 4th (back) page of wrapper	6 guineas.
For the half page of front wrapper	3 guineas.

Parts of a page are charged at a slightly higher rate.—Special quotations will be given for a series of three, six, or twelve months.

Teachers Advertising for Situations.

	s.	d.
20 words or under...	1	0
For every additional 10 words...	0	6

☞ **Portfolios, fitted with elastic bands, for preserving twelve numbers of 'The Practical Teacher,' may now be had. Price 2s. 6d.**

Handsome cases for binding yearly volumes, price 1s. 6d.

THE SCHOLASTIC SEWING MACHINE CO.

ESTABLISHED 1880.

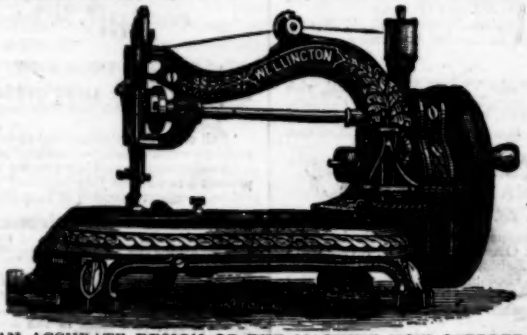
Highest and only Award, GOLD MEDAL, obtained at the Inventions Exhibition, 1885.

WE have great pleasure in offering to Fellow-Teachers a Superior £4 4s. HAND LOCKSTITCH MACHINE, by a First-Class Maker, for £2 10s. cash, or £3 on hire, carriage paid, repayable by Eight Monthly Instalments of 7s. 6d. each, or Three Quarterly Payments of £1 each.

This Machine is beautifully finished and ornamented in gold, and possesses the latest improvements to the present time. It has the Patent Shuttle and Spool Winder, which enables the person to wind the bobbins without the Machine being set in motion. It can be used for all sorts of Family Sewing, on any material, from Fine Muslin to Woollen Cloths. It will Hem, Tuck, Braid, Quilt, &c. The following extras are given free with each Machine:—Six bobbins, six needles, double hemmer, braider, quilter, two screw-drivers, oil can and oil straight, guide, thumb screw, and illustrated instruction book. This Machine is well made, and most complete in itself; and, in order that Teachers may thoroughly test its abilities, we purpose sending every one of them on Fourteen Days' Free Trial, and if satisfactory, we wish it to be perfectly understood that the cash or an instalment must be paid at the end of the fourteen days' free trial. If not satisfactory it can be returned, and we will pay the carriage both ways, and no charge whatever will be made.

It is earnestly requested that when Teachers are sending for a Machine they will write their full name and address distinctly, stating the nearest railway station, as this will prevent unnecessary delay.

50s. CASH. Carriage Paid.



60s. ON HIRE. Carriage Paid.

AN ACCURATE DESIGN OF THE MACHINE NOW OFFERED.

ABSTRACT OF TESTIMONIALS.

'The five "Wellington" Machines, both in appearance and simplicity, were far beyond my expectations. They have been thoroughly tested during the last three and a half years, and have proved highly satisfactory. They have been pronounced by my friends to be the prettiest Machines they ever saw.'—Mr. W. KENT, Schoolhouse, New Road, Grays, Essex.

'The three Machines (Bradbury) have worked well, and quite give satisfaction.'—The Dowager Lady G. CRAIG, Dean's Court, Wimborne, Dorset.

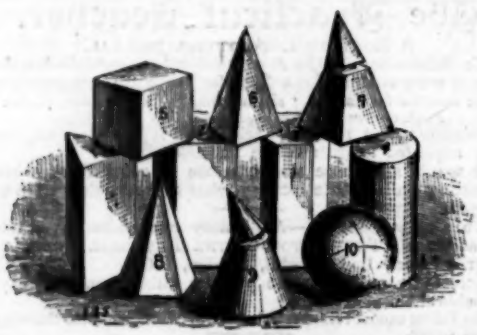
'We are very pleased with the three Machines (Bradbury's Wellington) purchased from you, and think they are very good and cheap.'—Miss PEASFIELD, Fairfield House, Duke Street, Chelmsford, Essex.

'I am well acquainted with the "Wellington" Machine, as several of my friends have them. It is an extremely pretty Machine, and does its work quickly, easily, and well. I am delighted with my Machine, and shall recommend your Company whenever I can.'—Miss THOMPSON, All Saints' School, Matlock Bank, Derbyshire.

'The Bradbury Machine I purchased from you gives great satisfaction to all who use it.'—Mrs. HAVILAND, The Rectory, Warbleton, Hawkhurst, Essex.

'My wife is very pleased with the three Machines (Bradbury) which we have purchased from you during the last two years.'—Mr. J. HARWOOD, Board School, Tongue End, Spalding.

Particulars, Testimonials, and Illustrated Lists from the MANAGER, 2, AVENUE ROAD, BOW, LONDON, E.



EYRE & SPOTTISWOODE,

Drawing Material Department.

The Student's Set of Ten Solid Pearwood Figures in Dove-tailed Sliding-lid Box. 10s. 6d. the Set.

The Drawing Class Set of Ten Larger Size Hollow Light wood Figures, with Stand, Complete in Case. Price, 42s.; in Polished Case, 45s.

CATALOGUES SENT FREE ON APPLICATION.

Great New Street, London, E.C.

MADE WITH BOILING WATER.

EPPS'S

GRATEFUL—COMFORTING.

COCOA

MADE WITH BOILING MILK.

PENLEY'S PATENT PEN GUIDE.

Invaluable for Teachers.

Prevents the fingers from becoming cramped.

Cost of an ordinary penholder.

'Nothing could be better for giving the Child a right start than this Penholder.'—*School Board Chronicle.*

SAMPLE SENT ON RECEIPT OF 3 STAMPS.

SOLD BY ALL STATIONERS.

E. WOLFF & SON, MANUFACTURERS, LONDON.

BORD'S PIANOS

42 SOUTHAMPTON ROW HOLBORN, LONDON W.C.

BRITISH